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NOISE ANALYSIS FOR DIGIT SLICING FFT

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In paper 8053F by Sharrif, Othman and Theong [1], a very interesting idea for the hardware implementation of an FFT using digit slicing is proposed and analysed. As usual, to avoid the overflow problem, a factor of 1/2 was used in the FFT butterflies [2].

A modification is made with respect to the standard algorithms with a factor of 1/2. The parameter a_m is introduced in the model. This parameter is taken to be equal to unity if the scaling by 1/2 is needed in any butterfly in the mth stage. Otherwise, a_m is taken to be zero and scaling is not done.

This idea, although it does not lead to easy implementation, is interesting and could give significant improvement. The parameter a_m is correctly used in the derivation of the mean-square error of quantisation noise. It is obtained from eqn. 22 of Reference 1:

$$\sigma_F^2 = 2\sigma_B^2 \sum_{m=0}^{v-1} [a_m(\tfrac{1}{2})^{(v-m-1)}] \tag{1}$$

$$a_m = \begin{cases} 0 \text{ if } m\text{th stage array is multiplied by } 1 \\ 1 \text{ if } m\text{th stage array is multiplied by } \tfrac{1}{2} \end{cases}$$

This fact that some stages are multiplied by 1/2 and some by 1 has been overlooked in Reference 1 when the mean-square magnitude of signal was calculated. The correct expression for the signal can be derived from the following considerations. The signal values that can be stored in the registers are in the interval from $-(2^{T-1}-1)$ to $(2^{T-1}-1)$. Assuming, as in Reference 1, that the signal is white with uniform distribution, we have that the mean-square magnitude of the input signal is $\sigma_x^2 = (2^{T-1}-1)^2/3 \cong 2^{(T-1)2}/3$. If we use the factor 1/2 when overflow occurs, then it is not necessary to divide the input signal any more.

If the factor 1/2 is not used (not needed) anywhere, then the mean-square magnitude of the output signal $X(k)$ is obtained simply summing the mean-square magnitude of input signals $x(n)$, because each of this signal components propagates to the output through a complex constant of unity magnitude. In this case we have $\sigma_y^2 = N\sigma_x^2$.

If the factor 1/2 is used in each stage, then the power in each stage is multiplied by 1/4 or, overall, each mean square magnitude of $x(n)$ by $1/2^{2v} = 1/N^2$. The mean-square magnitude of the output signal $X(k)$ is

$$\sigma_y^2 = \frac{2^{(T-1)2}}{3}\frac{1}{N^2}N = \frac{2^{(T-1)2}}{3N} = \sigma_x^2/N \tag{2}$$

This is formally the same result as in eqn. 25 of Reference 1.

If we use the parameter a_m (as is done in Reference 1) then the mean square magnitude of the output signal is

$$\sigma_y^2 = \frac{2^{(T-1)2}}{3} N \prod_{m=0}^{v-1} \frac{1}{2^{2a_m}} \tag{3}$$

By this expression, we take into account the influence of a_m on the output signal. If $a_m = 0$, the signal is propagated through the mth stage without attenuation, and if $a_m = 1$, i.e. a factor of 1/2 is used, then the mean-square magnitude is attenuated by 1/4. If $a_m = 1$ for all m, then eqns. 3 and 2 are identical and the result for the SNR given by eqn. 26 of Reference 1 is formally correct; but, if this is not the case, then the correct expression for the SNR is

$$SNR = \frac{\dfrac{2^{(T-1)2}}{3} N \displaystyle\prod_{m=0}^{v-1} \frac{1}{2^{2a_m}}}{2\sigma_B^2 \left\{\displaystyle\sum_k 2^{(p-1)2k} + \sum_j 2^{(-p)2j}\right\}\left\{\displaystyle\sum_{m=0}^{v-1} [a_m(\tfrac{1}{2})^{(v-m-1)}]\right\}} \tag{4}$$

Finally, we mention that, if any $a_m = 0$, then this is a more optimistic expectation for SNR than that obtained in Reference 1. The expression for the variance in eqn. 3 is always greater than or equal to the variance in eqn. 2. The equality holds only if $a_m = 1$ for all m.

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References

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This comment clarifies that the SNR as determined in the original paper has assumed that a factor of 1/2 had been used at every stage in order to avoid the overflow problem. The authors have appropriately shown that a more generalised formula for the SNR of a digit sliced FFT can be obtained, as shown in eqn. 4.

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