Moments of Multidimensional Polynomial FT

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Abstract— **Moments of the second order polynomial Fourier transform are considered. Relations between moments for various parameters in the polynomial FT are established. Estimation of multidimensional FM signals parameters based on the moments is proposed and compared with the phase derivative based estimator. Implementation issues of the proposed estimator are discussed.**

I. Introduction

The ambiguity function (AF) of 1D signals has been treated in details in [1]. Signal moments calculated based on the AF are used for determination of the optimal chirp-rate parameter in an FM signal. This approach can easily be related with the second order polynomial Fourier transform (FT) [2]. Moments of this polynomial Fourier transform for various chirp-rate parameters are not independent. All second order moments can be evaluated based on the signal and two second order polynomial FT moments calculated for different chirp-rate parameters [1]. A simple linear relationship can be established between the fractional Fourier transform (FRFT), recently reintroduced in the signal processing [3], and the second order polynomial FT. Moments of the FRFT are considered in [4]. It has been shown that all second order FRFT moments can be determined based on the signal and two other FRFT moments. Multidimensional (MD) signal moments, with their mutual relations, are studied in [5].

In this paper we consider second order moments of Q-dimensional signals and show that all of them can be determined based on $2Q^2 + Q$ second order moments. As a possible application of the proposed technique we consider estimation of the signal phase second partial derivatives.

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The approach from [1] is extended here in several directions: (a) Moments of MD signals are determined based on the MD AF; (b) Parameters of MD FM signals are estimated based on the MD polynomial FT moments; (c) Implementation issues (not studied in [1], even for 1D signal) are discussed.

II. Moments of MD signals

Consider an MD signal $f(\vec{t}\,)$, $\vec{t} = (t_1, t_2, ..., t_Q)$, with its FT given by $F(\vec{\omega}) = \int_{\vec{t}} f(\vec{t}\,)\exp(-j\vec{\omega}\vec{t}\,)d\vec{t}$, $\vec{\omega} = (\omega_1, \omega_2, ..., \omega_Q)$. The scalar multiplication of vectors is defined as $\vec{\omega}\vec{t} = \omega_1 t_1 + \omega_2 t_2 + ... + \omega_Q t_Q$. Second order moments of the signal and its FT are $w_{mn} = \int_{\vec{t}} t_m t_n |f(\vec{t}\,)|^2 d\vec{t}$ and $W_{mn} = (1/2\pi)^Q \int_{\vec{\omega}} \omega_m \omega_n |F(\vec{\omega})|^2 d\vec{\omega}$, $m, n = 1, ..., Q$. These moments, for $m = n$, can be considered as widths of the signal and the FT along the considered coordinate. The MD AF is defined as $A(\vec{\theta}, \vec{\tau}\,) = \int_{\vec{t}} f(\vec{t}+\vec{\tau}/2) f^*(\vec{t}-\vec{\tau}/2) e^{-j\vec{\theta}\vec{t}} d\vec{t} = (1/2\pi)^Q \int_{\vec{\omega}} F(\vec{\omega}+\vec{\theta}/2) F^*(\vec{\omega}-\vec{\theta}/2) e^{j\vec{\omega}\vec{\tau}} d\vec{\omega}$. Moments of signal and its FT can be expressed as $w_{mn} = -\partial^2 A(\vec{\theta}, \vec{\tau}\,)/\partial\theta_m \partial\theta_n|_{(\vec{\theta},\vec{\tau}\,)=(\vec{0},\vec{0}\,)}$ and $W_{mn} = -\partial^2 A(\vec{\theta}, \vec{\tau}\,)/\partial\tau_m \partial\tau_n|_{(\vec{\theta},\vec{\tau}\,)=(\vec{0},\vec{0}\,)}$, respectively. Mixed moments are defined by

$$\mu_{mn} =$$
$$= \frac{j}{2\pi}\int_{\vec{t}} t_m \left[f^*(\vec{t}\,)\frac{\partial f(\vec{t}\,)}{\partial t_n} - f(\vec{t}\,)\frac{\partial f^*(\vec{t}\,)}{\partial t_n} \right] d\vec{t}$$
$$= -\frac{j}{4(2\pi)^Q}\int_{\vec{\omega}} \omega_n$$
$$\times \left[F^*(\vec{\omega})\frac{\partial F(\vec{\omega})}{\partial \omega_m} - F(\vec{\omega})\frac{\partial F^*(\vec{\omega})}{\partial \omega_m} \right] d\vec{\omega} \quad (1)$$

or, by using the AF, as $\mu_{mn} = -\partial^2 A(\vec{\theta}, \vec{\tau}\,)/ \partial\theta_m \partial\tau_n|_{(\vec{\theta},\vec{\tau}\,)=(\vec{0},\vec{0}\,)}$.

Consider an auxiliary signal $f_P(\vec{t}\,)$ producing a linear coordinate transformation of the

AF $A(\vec{\theta},\vec{\tau})$, i.e., $A_P(\vec{\theta},\vec{\tau}) = A(\mathbf{A}\vec{\theta}+\mathbf{B}\vec{\tau},\mathbf{C}\vec{\theta}+\mathbf{D}\vec{\tau})$, where $\mathbf{A} = [a_{mn}, m,n = 1,...,Q]$, $\mathbf{B} = [b_{mn}, m,n = 1,...,Q]$, $\mathbf{C} = [c_{mn}, m,n = 1,...,Q]$, $\mathbf{D} = [d_{mn}, m,n = 1,...,Q]$[1]. Moments of the signal $f_P(\vec{t})$ can be expressed by using the corresponding moments of $f(\vec{t})$, as

$$w^P_{mn} = \sum_{k=1}^{Q}\sum_{l=1}^{Q}[w_{kl}a_{km}a_{ln}+ \mu_{kl}a_{km}c_{ln} + \mu_{lk}c_{km}a_{ln} + W_{kl}c_{km}c_{ln}] \tag{2}$$

$$\mu^P_{mn} = \sum_{k=1}^{Q}\sum_{l=1}^{Q}[w_{kl}a_{km}b_{ln}+ \mu_{kl}a_{km}d_{ln} + \mu_{lk}c_{km}b_{ln} + W_{kl}c_{km}d_{ln}] \tag{3}$$

$$W^P_{mn} = \sum_{k=1}^{Q}\sum_{l=1}^{Q}[w_{kl}b_{km}b_{ln}+ \mu_{lk}d_{km}b_{ln} + \mu_{kl}b_{km}d_{ln} + W_{kl}d_{km}d_{ln}]. \tag{4}$$

Let the auxiliary signal be of the form $f_P(\vec{t}) = f(\vec{t})\exp(-j\sum_{r=1}^{Q}\rho_{rr}t_r^2/2 - j\sum_{r=1}^{Q}\sum_{p=1,r>p}^{Q}\rho_{rp}t_rt_p)$. Note that its FT can be considered as a generalized polynomial FT [6]. Its AF reads

$$A_P(\vec{\theta},\vec{\tau}) = A(\theta_1 + \sum_{r=1}^{Q}\rho_{1r}\tau_r, ..., \theta_k + \sum_{r=1}^{Q}\rho_{kr}\tau_r, ..., \theta_Q + \sum_{r=1}^{Q}\rho_{Qr}\tau_r; \tau_1, ..., \tau_Q).$$

The corresponding coordinate transformation matrices are $\mathbf{A} = \mathbf{I}_Q$, $\mathbf{B} = \mathbf{R} = [\rho_{mn}, m,n = 1,2,...,Q]$ (note that $\mathbf{R} = \mathbf{R}^T$, since $\rho_{mn} = \rho_{nm}$), $\mathbf{C} = \mathbf{0}_Q$, $\mathbf{D} = \mathbf{I}_Q$, where $\mathbf{I}_Q$ is a $Q\times Q$ identity matrix, and $\mathbf{0}_Q$ is a $Q\times Q$ zero matrix. Moments of the signal $f_P(\vec{t})$ are equal to $w^P_{mn} = w_{mn}$, $\mu^P_{mn} = \sum_{l=1}^{Q} w_{ml}\rho_{ln} + \mu_{mn}$, and

$$W^P_{mn} = \sum_{k=1}^{Q}\sum_{l=1}^{Q} w_{kl}\rho_{km}\rho_{ln} + \sum_{k=1}^{Q}\mu_{lm}\rho_{ln} + \sum_{k=1}^{Q}\mu_{kn}\rho_{km} + W_{mn}. \tag{5}$$

[1]Determination of a generalized signal transform, $f_P(\vec{t}) = T\{f(\vec{t})\}$, that produces linear coordinate transformation of the AF, $A_P(\vec{\theta},\vec{\tau})$, is not of our particular interest. Thus, due to the lack of space, this transform is not considered in our derivations.

Spectral moments W^P_{mm} represent widths of the polynomial FT along the corresponding direction. Assume that the original signal can be written as $f(\vec{t}) = A(\vec{t})\exp(j\phi(\vec{t}))$, where the signal amplitude is slow-varying as compared to the signal phase[2], $|\partial A(\vec{t})/\partial t_m| \ll |\partial\phi(\vec{t})/\partial t_m|$, $m = 1,...,Q$. Values of ρ_{mn}, $m,n = 1,...,Q$, that minimize the moments W^P_{mm} are estimates of the signal phase partial derivatives, $\rho_{mn} \approx \partial^2\phi(\vec{t})/\partial t_m\partial t_n$ [1]. Minimization of W^P_{mm}, with respect to $\rho_{\alpha\beta}$, by using $\partial W^P_{mm}/\partial\rho_{\alpha\beta} = 0$, yields

$$\sum_{k=1}^{Q} w_{k\alpha}\rho_{km} = -\mu_{\alpha m},$$

$$\alpha = 1,...,Q, m = 1,...,Q$$

$$\begin{bmatrix} w_{11} & w_{21} & \dots & w_{Q1} \\ w_{12} & w_{22} & \dots & w_{Q_2} \\ \dots & \dots & \dots & \dots \\ w_{1Q} & w_{2Q} & \dots & w_{QQ} \end{bmatrix} \begin{bmatrix} \rho_{1m} \\ \rho_{2m} \\ \dots \\ \rho_{Qm} \end{bmatrix} = = -\begin{bmatrix} \mu_{1m} \\ \mu_{2m} \\ \dots \\ \mu_{Qm} \end{bmatrix}, \tag{6}$$

$m = 1,...,Q$. Since $\rho_{mn} = \rho_{nm}$, (6) represents a system of Q^2 equations with $Q(Q+1)/2$ unknowns. A way to solve this system, with a discussion of the implementation issues, is given in the next section.

III. Implementation issues:

1. Discretized limited-space signals, sampled according to the Nyquist rate, and the discrete-spatial frequency MD FT are considered in realizations. We assume that errors introduced by discretization are negligible, as compared with other possible sources of disturbances.

[2]Since the amplitude and phase functions of a real-valued FM signal cannot be defined uniquely, an appropriate definition of the complex-valued signals is still important research topic [8]. We assume that a real-valued signal satisfies conditions given in [9] and that the complex-valued signal is formed as its analytical extension.

2. Solving system of equations (6) requires determination of the mixed moments. Calculation of these moments is based on a differentiation of the signal or its spectra (1). Differentiation can be sensitive, even to the quantization error. Therefore, instead of a direct calculation of the mixed moments we will calculate moments of an auxiliary signal $f_\alpha(\vec{t}\,) = f(\vec{t}\,)\exp(-jt_\alpha^2/2)$. Its spectral moments are given as:

$$W_{\alpha\alpha}^{\alpha} = w_{\alpha\alpha} + 2\mu_{\alpha\alpha} + W_{\alpha\alpha},$$

$$W_{\alpha,\alpha\neq n}^{\alpha} = \mu_{\alpha n} + W_{\alpha n} \quad \text{for } n \neq \alpha.$$

Thus, based on the signal's and spectral moments, all mixed moments can be calculated as $\mu_{\alpha\alpha} = (W_{\alpha\alpha}^{\alpha} - w_{\alpha\alpha} - W_{\alpha\alpha})/2$ and $\mu_{\alpha n} = W_{\alpha n}^{\alpha} - W_{\alpha n}$, for $\alpha \neq n$. In this way, the differentiation of signal or its FT is avoided. Determination of the mixed moments requires calculation of Q^2 moments, in addition to $Q(Q+1)/2$ signal moments w_{mn}, and $Q(Q+1)/2$ spectral moments, i.e., $2Q^2+Q$ moments in total. Also, it is required to calculate Q additional MD FTs of the signals $f_\alpha(\vec{t}\,)$, i.e., $(Q+1)$ in total.

3. Consider the 1D signal case. The chirp rate parameter can be estimated as: $w_{11}\rho_{11} = \mu_{11}$, i.e., $\rho_{11} = \mu_{11}/w_{11}$, where $\mu_{11} = (W_{11}^1 - w_{11} - W_{11})/2$, and $\rho_{11} = (W_{11}^1 - W_{11})/2w_{11} - 1/2$.

4. For a 2D signal we obtain four equations with three unknowns:

$$\begin{bmatrix} w_{11} & w_{21} & 0 \\ w_{12} & w_{22} & 0 \\ 0 & w_{11} & w_{21} \\ 0 & w_{12} & w_{22} \end{bmatrix} \begin{bmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{22} \end{bmatrix} = - \begin{bmatrix} \mu_{11} \\ \mu_{21} \\ \mu_{12} \\ \mu_{22} \end{bmatrix}. \tag{7}$$

Mixed moments can be calculated as $\mu_{11} = (W_{11}^1 - w_{11} - W_{11})/2$, $\mu_{22} = (W_{22}^2 - w_{22} - W_{22})/2$, $\mu_{12} = W_{12}^1 - W_{12}$, $\mu_{21} = W_{12}^2 - W_{12}$.

5. Problem of larger number of equations than unknowns (see (6) and (7)) can be solved in several ways. Here we use the Moore-Penrose pseudo-inverse (denoted by #) as

$$\begin{bmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{22} \end{bmatrix} = - \begin{bmatrix} w_{11} & w_{21} & 0 \\ w_{12} & w_{22} & 0 \\ 0 & w_{11} & w_{21} \\ 0 & w_{12} & w_{22} \end{bmatrix}^{\#} \begin{bmatrix} \mu_{11} \\ \mu_{21} \\ \mu_{12} \\ \mu_{22} \end{bmatrix}. \tag{8}$$

Similar form follows in the case of signals with $Q > 2$. Since our realization is discrete-space and discrete-spatial frequency, bounded in space and spatial-frequency domain, we cannot be sure that various subsystems of equations (6) will produce the same solution for ρ_{mn}. Therefore, implementation involving the pseudo-inverse (resulting in the mean squared error solution) is better than other possible procedures based on subsystems of (7).

6. Consider an 1D noisy signal $x(t) = f(t) + \nu(t)$, $E\{\nu(t)\} = 0$ and $E\{|\nu(t)|^2\} = \sigma^2$. Mathematical expectation is $E\{W_{11,x}^1 - W_{11,x}\} = W_{11,f}^1 - W_{11,f}$, where additional index denotes the considered signal. However, estimate of the signal moment is biased, since $E\{w_{11,x}\} = w_{11,f} + g_1\sigma^2$, where $g_1 = \int_{\vec{t}} t_1^2 d\vec{t}$ (g_1 is bounded because the signal is considered within the limited space). Then, the estimate of ρ_{11} is also biased, namely $|E\{\rho_{11,x}\}| < |\rho_{11,f}|$. Note that, in practice, there are numerous estimators of the noise parameters. If we can estimate noise variance $\hat{\sigma}^2$, the estimate of ρ_{11} can be calculated as $\hat{\rho}_{11} \cong (W_{11,x}^1 - W_{11,x})/2(w_{11,x} - g_1\hat{\sigma}^2) - 1/2$. Similarly, for $Q > 1$ all $w_{\alpha\alpha}$ should be decreased for $g_\alpha\hat{\sigma}^2$, $g_\alpha = \int_{\vec{t}} t_\alpha^2 d\vec{t}$. Further improvement can be achieved by applying a localized evaluation of the moments.

IV. Numerical example

Consider a 2D signal, $Q = 2$, $f(t_1,t_2) = A\exp(jat_1^2/2 + jbt_1t_2 + jct_2^2/2 + jd_1t_1 + jd_2t_2 + j\phi)$. We set parameters d_1, d_2 and ϕ to zero, and parameter A to one. Signal is considered within the range $[t_1,t_2] = [-1,1) \times [-1,1)$, with the sampling rate $1/64 \times 1/64$ (number of samples is 128×128). Gaussian noise environment has been considered, with the varying signal-to-noise ratio $SNR \in [-40,40]$dB. We compared the proposed estimator of parameters (a,b,c) (with the correction of moments $w_{\alpha\alpha}$) with the phase derivative (PD) based estimator [10], [11], [12]. Several comments on the PD based estimator are given in the Appendix. Three realizations of the PD based estimator are considered: (1) without interpolation in the frequency domain; (2) and (3) with interpolation with factors 2 and 4 along each coordinate. The interpolation is required in order to decrease the error caused by frequency discretization. In order to compare these estimators on a fair basis, we randomly selected values of parameters (a, b, c) in each trial. For each noise variance from the range, we performed 1000 trials. The mean squared error in the parameters estimation, obtained by using these estimators, is depicted in Fig.1. The region of high signal to noise ratio has been magnified to the right[3]. It can be seen that for $SNR > 13$dB the moments based estimator outperforms the PD based estimator without interpolation, as well as the PD estimator with interpolation factor 2. The PD based estimators with interpolation factor 4 produces the highest accuracy. One trial in the case of the moments based estimator requires 6.47×10^6 floating point operations (evaluated by using the MATLAB flops function), while for the PD based estimator without interpolation 2.60×10^6 operations are needed, and 7.24×10^6 and 2.76×10^7 operations for interpolation with factors 2 and 4, respectively. Note that in numerous applications signal is embedded in a moderate noise. In that case the proposed estimator could be, at the same time, more accurate and more efficient than the PD based counterpart. However, the PD based estimator can improve results by interpolation with a larger factor. In that case calculation demands would be increased.

[3]Lower bound of the MSE in the case of PD based estimators is determined by $(\Delta\omega)^2/12K^2$, where $\Delta\omega$ is the frequency discretization step, while K is the interpolation factor. Similar bound exists in the case of the proposed estimator. It is caused by evaluation of moments based on the discrete-time and discrete-spatial frequency observations, (see Fig.1). The PD based estimators for small SNR, $SNR < -13$dB, assume constant value since for high noise the probability distribution function of the estimation error is uniform (the estimate is randomly selected from the whole considered range in the FFT).

V. Conclusion

Moments of the multidimensional signals have been considered. It has been shown that all the second order moments can be determined based on $2Q^2 + Q$ different moments of the local polynomial FT. Useful relations for calculation of the moments are derived. The procedure for estimation of the chirp-rate parameters of multidimensional signals, based on the moments of the polynomial Fourier transform, has been presented. Implementation issues are discussed. Accuracy and efficiency of the proposed estimator are compared with the phase derivative based one.

VI. Appendix: Phase derivative based estimator

Consider a signal embedded in a white Gaussian noise $x(t_1, t_2) = f(t_1, t_2) + \nu(t_1, t_2) = A\exp(jat_1^2/2 + jbt_1t_2 + jct_2^2/2 + jd_1t_1 + jd_2t_2 + j\phi) + \nu(t_1, t_2)$. The signal auto-correlation functions are:

$$\begin{aligned} r_{x1}(t_1, t_2; \tau_1) &= x(t_1 + \tau_1, t_2)x^*(t_1, t_2) = \\ f(t_1 + \tau_1, t_2)f^*(t_1, t_2) &+ \tilde{r}_{n1}(t_1, t_2; \tau_1) = \\ &r_{f1}(t_1, t_2; \tau_1) + \tilde{r}_{n1}(t_1, t_2; \tau_1) \qquad (9) \\ r_{x2}(t_1, t_2; \tau_2) &= x(t_1, t_2 + \tau_2)x^*(t_1, t_2) = \\ &r_{f2}(t_1, t_2; \tau_2) + \tilde{r}_{n2}(t_1, t_2; \tau_2), \end{aligned}$$

where τ_1 and τ_2 are known quantities, while $r_{f1}(t_1, t_2; \tau_1)$ and $r_{f2}(t_1, t_2; \tau_2)$ are the 2D complex sinusoids:

$$r_{f1}(t_1, t_2; \tau_1) =$$
$$= A^2 \exp(jat_1\tau_1 + jbt_2\tau_1 + ja\tau_1^2/2 + j\omega_1\tau_1)$$
$$r_{f2}(t_1, t_2; \tau_2) =$$
$$= A^2 \exp(jbt_1\tau_2 + jct_2\tau_2 + jc\tau_2^2/2 + j\omega_2\tau_2).$$

Their 2D FTs are ideally concentrated at the frequencies $(a\tau_1, b\tau_1)$ and $(b\tau_2, c\tau_2)$, respectively. Therefore, the unknown chirp-rate parameters can be estimated as:

$$(\hat{a}, \hat{b}') = \frac{1}{\tau_1} \arg \max_{(\omega_1, \omega_2)} |R_{x1}(\omega_1, \omega_2; \tau_1)|$$

$$(\hat{b}'', \hat{c}) = \frac{1}{\tau_2} \arg \max_{(\omega_1, \omega_2)} |R_{x2}(\omega_1, \omega_2; \tau_2)|$$

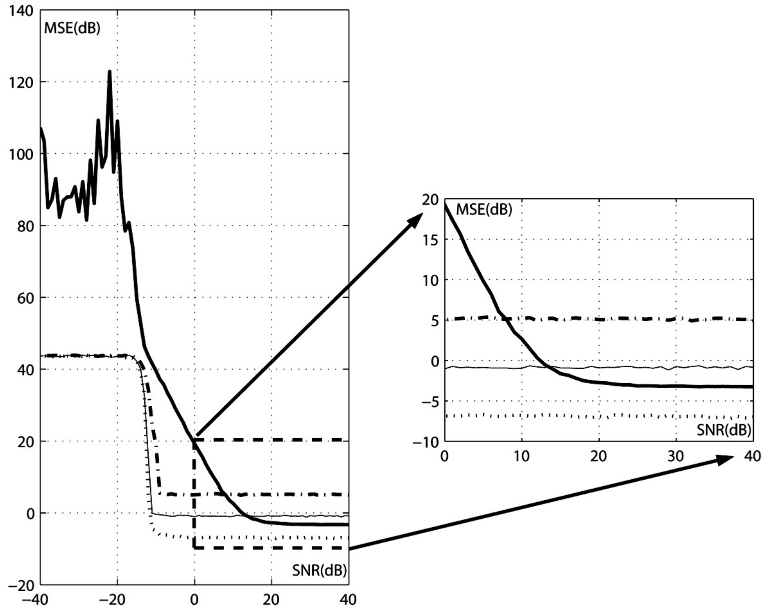

Fig. 1. MSE in the polynomial phase signals parameter estimation. Thick line - moments based estimator; thin line - PD based estimator without interpolation; dash-dot line - PD estimator with interpolation with factor 2; dotted line - PD estimator with interpolation with factor 4. Region of high SNR is magnified on the right.

where $R_{xi}(\omega_1, \omega_2; \tau_i)$, $i = 1, 2$, are 2D FTs of $r_{xi}(t_1, t_2; \tau_i)$, $i = 1, 2$. We set the estimate of b as $\hat{b} = (\hat{b}' + \hat{b}'')/2$. The frequency discretization in the 2D FT calculation, introduced by applying the FFT algorithms, produces error in the algorithm that can be higher than the noise influenced errors. Therefore, an interpolation of 2D FT is usually performed (by zero-padding of $r_{xi}(t_1, t_2; \tau_i)$, $i = 1, 2$ in space domain) in order to decrease this error. The interpolation increases the calculation complexity of the algorithm.

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