Adaptive Windowed Fourier Transform

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Abstract— **Adaptive Fourier transform with a data-driven window function is proposed in the paper. The algorithm, based on the confidence intervals intersection, is used for determination of the window length. It produces an adaptive, close to optimal, bias-to-variance trade-off. Generalization for the case of time-varying signals is given. Numerical examples and statistical analysis illustrate and confirm the presented theory.**

I. Introduction

In biased estimators the bias value is usually proportional to an estimator parameter. In many cases the estimation variance caused by a stochastic influence is inversely proportional to the same parameter. In these cases there exists a bias-to-variance trade-off which results in the minimal mean squared error (MSE). The MSE is defined as a sum of the squared bias and the variance.

The bias usually cannot be determined in advance since it depends on the estimated value and its derivatives. Thus, the optimal estimation parameter, producing a bias-to-variance trade-off, cannot be used in practical realizations. In order to produce an estimate of this parameter, as close as possible to the optimal one, the non-parametric algorithm is proposed [1]. It is based on the intersection of the confidence intervals (ICI). Algorithm origins are in non-parametric regression [2]. The algorithm is applied on the instantaneous frequency (IF) estimation based on the Wigner distribution and on other time-frequency (TF) distributions [1], [3]. Other algorithm applications are: signal and image filtering [4], [5], signal denoising [6], [7], determination of the TF distributions' values [8], variable-step LMS algorithms, and the direction-of-arrival estimation [9].

In this paper, the algorithm is applied on the calculation of the Fourier transform (FT). Namely, the bias-to-variance trade-off exists in the case of the FT. This fact is noted in the famous book [10], where optimal window width in the FT is derived. It cannot be used in calculations since the bias of the FT depends on its unknown derivatives. Note that an optimal smoothed periodogram, based on the bias-to-variance trade-off, is proposed in [11]. However, method for derivation, background theory, and application field of the approach from [11], is quite different than the approach proposed in this paper.

Signal Processing, Vol.82, No.1, Jan.2003.

The paper is organized as follows. Optimal window width in the case of the FT is derived in Section II. Algorithm for the optimal window width estimation is given in Section III. The FT produced with estimate of the optimal window width is called adaptive windowed Fourier transform. Numerical examples and statistical analysis are presented in Section IV.

II. Optimal Window Width in the Fourier Transform

Consider signal $f(n)$ corrupted by a white Gaussian noise $\nu(n)$ with the variance σ^2, $\mathcal{N}(0,\sigma^2)$. The goal is to estimate the FT of signal $f(n)$:

$$F(\omega) = \sum_{n=0}^{N-1} f(n)e^{-j\omega n}, \tag{1}$$

based on the noisy signal $x(n)$ observations. Here, the estimation will be performed by using the windowed FT:

$$X(\omega) = \sum_{n=0}^{N-1} x(n)w(n)e^{-j\omega n}. \tag{2}$$

Variance of the windowed FT (2):

$$\sigma^2(w) = E\{|X(\omega)|^2\} - |E\{X(\omega)\}|^2 = \sigma^2 \sum_{n=0}^{N-1} w^2(n) \tag{3}$$

is window dependent. For a Hanning window, that will be used in numerical examples, the variance is:

$$\sigma^2(N) = \frac{3}{8}\sigma^2 N. \qquad (4)$$

It is proportional to the window width. Similar conclusion holds for all window forms used in practice.

The bias and variance of the FT can be considered separately since

$$E\{X(\omega)\} = \sum_{n=0}^{N-1} f(n)w(n)e^{-j\omega n}. \qquad (5)$$

The bias can be derived from:

$$E\{X(\omega)\} = \sum_{n=0}^{N-1} f(n)w(n)e^{-j\omega n}$$
$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\theta)F(\omega-\theta)d\theta. \qquad (6)$$

By expanding the FT $F(\omega)$ into the Taylor series around ω:

$$F(\omega-\theta) = F(\omega) + \sum_{k=1}^{\infty}\frac{(-1)^k F^{(k)}(\omega)\theta^k}{k!}, \qquad (7)$$

expression (6), for even and symmetric window function, can be written as:

$$E\{X(\omega)\} = F(\omega) +$$
$$+\frac{1}{2\pi}\sum_{k=1}^{\infty}\frac{F^{(2k)}(\omega)}{(2k)!}\int_{-\pi}^{\pi} W(\theta)\theta^{2k}d\theta, \qquad (8)$$

where the second term on the right hand side of (8) represents the bias. By neglecting the higher-order derivatives of the FT, $F^{(k)}(\omega) = 0$ for $k > 2$, we get

$$E\{X(\omega)\} \approx F(\omega) + \frac{F''(\omega)}{4\pi}\int_{-\pi}^{\pi} W(\theta)\theta^2 d\theta =$$
$$= F(\omega) - \frac{F''(\omega)}{2}w''(0). \qquad (9)$$

For Hanning window holds $w''(0) = 2\pi^2/N^2$, and the bias is:

$$bias\{X(\omega)\} = E\{X(\omega)\} - F(\omega) = -\frac{\pi^2}{N^2}F''(\omega). \qquad (10)$$

The MSE of the FT estimate is:

$$MSE(\omega; N) = bias^2\{X(\omega)\} + \sigma^2(N)$$
$$= \frac{\pi^4}{N^4}[F''(\omega)]^2 + \frac{3}{8}\sigma^2 N. \qquad (11)$$

The optimal window width, producing minimal MSE value, follows from

$$\frac{\partial MSE(\omega; N)}{\partial N}|_{N=N_{opt}(\omega)} = 0,$$
$$-\frac{4\pi^4}{N^5}[F''(\omega)]^2 + \frac{3}{8}\sigma^2 = 0, \qquad (12)$$

resulting in:

$$N_{opt}(\omega) = \sqrt[5]{\frac{32\pi^4[F''(\omega)]^2}{3\sigma^2}}. \qquad (13)$$

Similar relationship is derived in [10]. However, the optimal window width determination was not discussed. As it can be seen, expression (13) contains the unknown FT derivative, $F''(\omega)$. Thus, it cannot be used for a direct 'optimal' FT calculation. Next we will present a method for the parameter $N_{opt}(\omega)$ estimation without knowing the value of $F''(\omega)$.

III. Non-parametric Algorithm

An adaptive algorithm, based on the ICI rule, for the IF estimation has been developed in [1]. It can be used when the MSE, in terms of parameter N, is of the form:

$$MSE(\omega, N) = BN^n + \frac{A(\omega)}{N^m}, \qquad (14)$$

where

$$\sigma^2(N) = BN^n, \qquad (15)$$

is the estimation variance, while the second term in (14) represents the squared bias

$$bias^2(\omega, N) = A(\omega)/N^m. \qquad (16)$$

The MSE is a relatively slow varying function around its minimum [12]. It means that satisfactory results can be obtained by considering a set with relatively small number of the parameter N values. In the IF estimation case, the same as in our case of the adaptive FT calculation, the parameter N is the window width. For implementation of the FFT algorithms it is

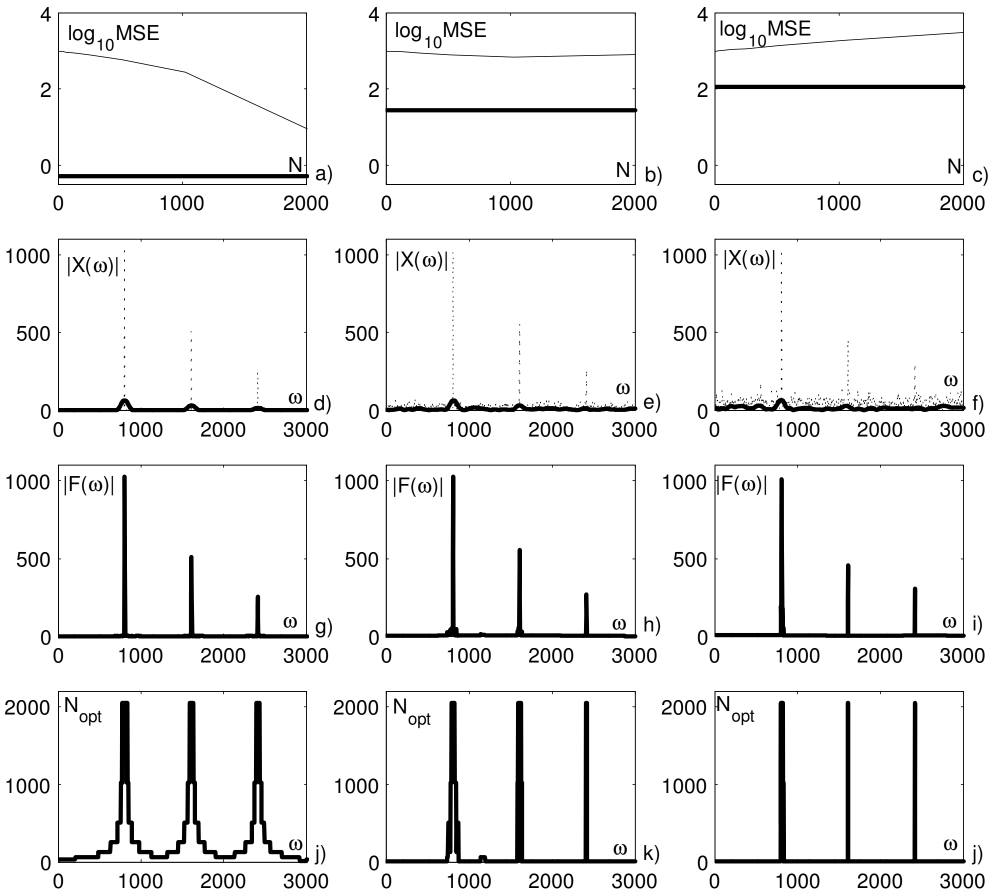

Fig. 1. Adaptive FT for a sum of three complex sinusoids: First column - $\sigma = 0.1$; second column - $\sigma = 1$; third column - $\sigma = 2$. First row - logarithm of the MSE as function of the window width, thick line represents MSE for the adaptive algorithm; second row - FTs with the constant window width: $N = 2048$ - dashed line; $N = 128$ - thick line; third row - adaptive FT; fourth row - adaptive window width.

suitable that the windows have dyadic widths. Define a set $\mathbf{N}$ with such window widths:

$$\mathbf{N} = \{N^{(s)} | N^{(s)} = 2N^{(s-1)}, s = 1, 2, ..., J\}. \tag{17}$$

staring with a very narrow window $N^{(1)}$, up to a very wide window $N^{(J)}$.

Let $F(\omega)$ be the true value of the estimated variable (true FT). The estimates obtained with parameters from set $\mathbf{N}$ are denoted by $\hat{F}_{N^{(s)}}(\omega)$, $s = 1, 2, ..., J$. For an estimate $\hat{F}_{N^{(s)}}(\omega)$ as a random variable, with bias $bias(\omega, N^{(s)})$ and standard deviation $\sigma(N^{(s)})$, like for any other random variable, the following inequality can be written as

$$|F(\omega) - (\hat{F}_{N^{(s)}}(\omega) - bias(\omega, N^{(s)}))| \leq \kappa\sigma(N^{(s)}). \tag{18}$$

This inequality is holds with probability $P(\kappa)$. We will assume that parameter κ is such that the probability $P(\kappa)$ is very close to 1.

The confidence interval around the estimate is

$$D_s \in [\hat{F}_{N^{(s)}}(\omega) - (\kappa + \Delta\kappa)\sigma(N^{(s)}),$$
$$\hat{F}_{N^{(s)}}(\omega) + (\kappa + \Delta\kappa)\sigma(N^{(s)})]. \tag{19}$$

Values κ and $\Delta\kappa$ determine algorithm accuracy and they depend on m and n in (14).

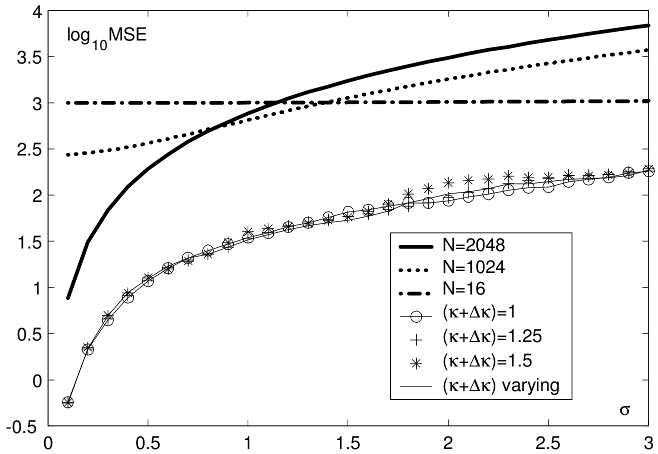

Fig. 2. MSE for various constant window widths, and for adaptive algorithm with various $(\kappa+\Delta\kappa)$ for the case of a sum of three complex sinusoids.

Details on determination of κ and $\Delta\kappa$ can be found in [13].

Basic idea of the algorithm: When the bias is small, i.e., $F''(\omega)/N^2$ can be neglected, then the confidence intervals intersect since $\hat{F}_{N^{(s)}}(\omega)$ are unbiased estimates. Therefore, the true FT $F(\omega)$ belongs to these intervals with probability $P(\kappa+\Delta\kappa) \to 1$. This is always true for extremely wide windows. For narrow windows the bias is dominant with respect to the variance and the confidence intervals do not intersect. The optimal window is somewhere between these two extreme cases, for finite and nonzero values of $F''(\omega)$ and σ^2. It can be shown that the parameters κ and $\Delta\kappa$ can be determined in such a way that the intersection of the confidence intervals $(D_s \cap D_{s-1})$ works as an indicator of the window width close to the optimal one, for a given frequency ω. A detailed analysis, which may be applied here, in a straightforward manner, can be found in [13].

It has been derived that the values of κ and $\Delta\kappa$ should satisfy:

$$\Delta\kappa = \sqrt{\frac{m}{n}} 2^{m/2} \frac{2^{n/2}-1}{2^{m/2}+1}, \tag{20}$$

$$\kappa < \sqrt{\frac{m}{n}} 2^{m/2-1} \frac{2^{n/2}-1}{2^{m/2}+1} (2^{(m+n)/2}-1). \tag{21}$$

The adaptive value of N is determined as the smallest $N^{(s)} \in \mathbf{N}$ where two consecutive confidence intervals intersect:

$$|\hat{F}_{N^{(s)}}(\omega) - \hat{F}_{N^{(s-1)}}(\omega)| \leq$$

$$(\kappa+\Delta\kappa)[\sigma(N^{(s)}) + \sigma(N^{(s-1)})]. \tag{22}$$

Note that in practical considerations the exact value of the estimation standard deviation $\sigma(N^{(s)})$ is not known. Therefore, it is necessary to estimate the standard deviation $\hat{\sigma}(N^{(s)})$. Estimation procedure depends on the particular problem nature.

A. *Algorithm*

Based on the facts from this section, the following algorithm for the adaptive FT determination can be defined.

1. Consider a set of the window widths $\mathbf{N} = \{N^{(s)} | N^{(s)} = 2N^{(s-1)}, s = 1, 2, ..., J\}$.

2. Calculate the FT for each window from the set $\mathbf{N}$, $X_s(\omega)$, $s = 1, 2, ..., J$.

3. The initial guess is the FT produced with the widest window from the set $\mathbf{N}$, $X_J(\omega)$. Note that this FT has the smallest bias from all FTs obtained with windows from the considered set.

4. The adaptive FT, $\hat{F}(\omega)$, for a considered ω, is the one produced with the narrowest window from the set $\mathbf{N}$ where the following

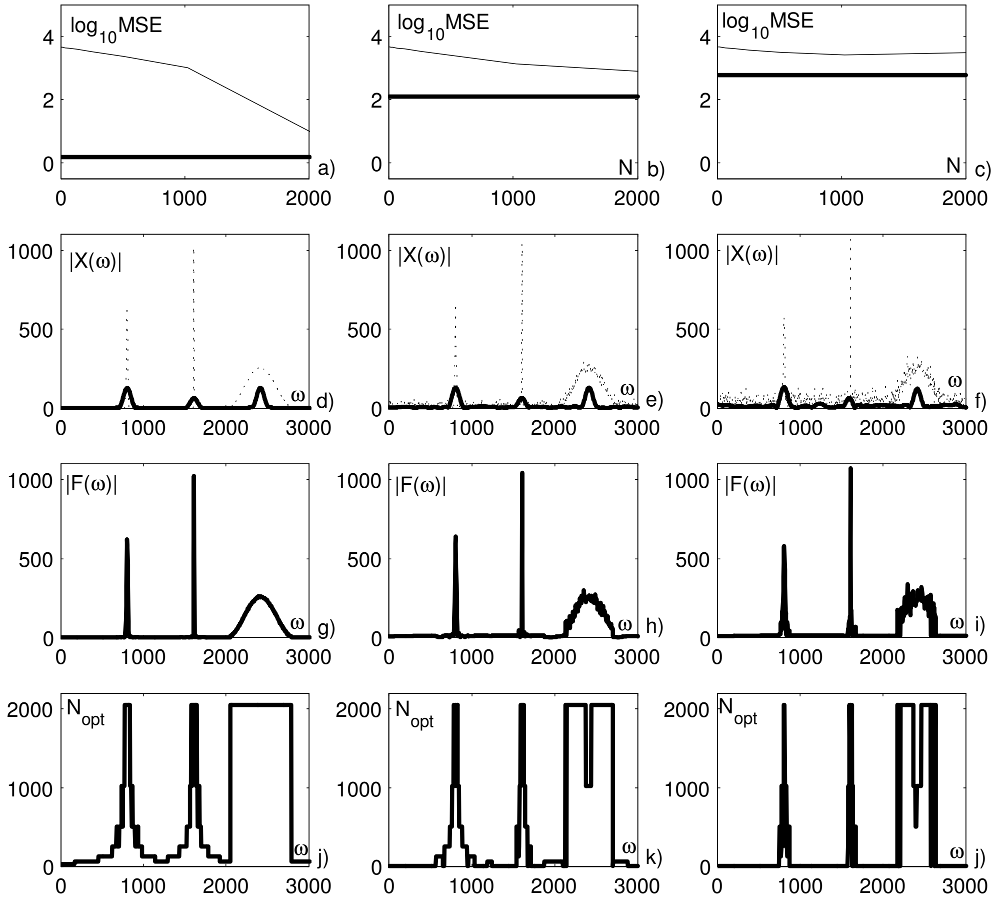

Fig. 3. Adaptive FT for a sum of complex sinusoid, dumped complex sinusoid and linear FM signal: First column - $\sigma = 0.1$; second column - $\sigma = 1$; third column - $\sigma = 2$. First row - logarithm of the MSE as the function of the window width, thick line represents MSE for the adaptive algorithm; second row - FTs with the constant window width: $N = 2048$ - dashed line; $N = 128$ - thick line; third row - adaptive FT; fourth row - adaptive window width.

inequality still holds:

$$|X_s(\omega) - X_{s-1}(\omega)| \leq$$

$$(\kappa + \Delta\kappa)[\sigma(N^{(s)}) + \sigma(N^{(s-1)})], \tag{23}$$

$$\hat{F}(\omega) = X_s(\omega). \tag{24}$$

5. The adaptive (an estimate of the optimal) window width is:

$$\hat{N}(\omega) = N^{(s)}. \tag{25}$$

Comments on the Algorithm:

a) Model described by equations in (11) and (14) has parameters $m = 4$ and $n = 1$, that produce $\Delta\kappa = 0.6627$, $\kappa < 1.5431$ (20) and (21). Thus, values of $(\kappa + \Delta\kappa)$ can be chosen within the interval $(\kappa + \Delta\kappa) \in [0.6627, 2.2059]$. For smaller $(\kappa + \Delta\kappa)$, probability that algorithm produces narrower window is smaller while, for larger $(\kappa + \Delta\kappa)$, probability that algorithm takes narrower window is higher. Therefore, higher $(\kappa + \Delta\kappa)$ gives better noise suppressing, but at the same time it can suppress weak signal components. Smaller $(\kappa + \Delta\kappa)$ exhibits worse performance for noise suppressing, but it is better for keeping weak signal components. Weak components can be essential for speech signals analysis where they are very important for the audio quality. Thus, the choice of $(\kappa + \Delta\kappa)$ depends on the consid-

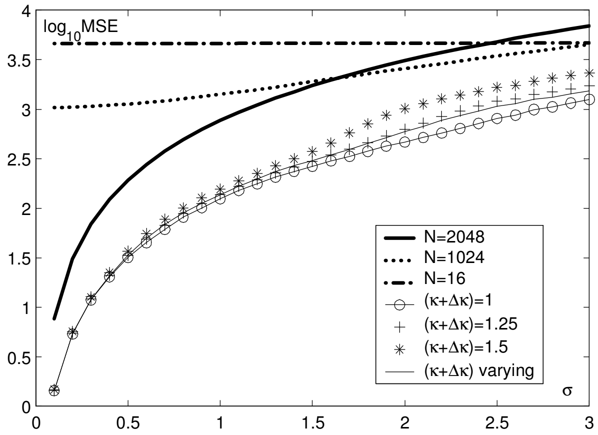

Fig. 4. MSE for various constant window widths and for adaptive algorithm with various $(\kappa+\Delta\kappa)$ for the case of the sum of complex sinusoids, dumped complex sinusoid and linear FM signal.

ered application. Its influence will be analyzed within the numerical examples.

b) The unknown standard deviation $\sigma(N^{(s)})$ of the FT (23) should be estimated. It can be done on numerous ways. For the FM signal it can be done as in [1], while for the speech signal it can be done by considering the speech signal pause, as it is described in [14].

c) The adaptive FT for the Hanning window can be written as:

$$\hat{X}(\omega)=\sum_{n=0}^{N-1} x(n)w(n,\omega)e^{-j\omega n} \tag{26}$$

where:

$$w(n,\omega)=\frac{1}{2}\left(1-\cos\frac{2\pi n}{\hat{N}(\omega)}\right). \tag{27}$$

B. Application in Time-Frequency Analysis

For signals with time variations of the spectral content, the short-time Fourier transform (STFT):

$$STFT(n,\omega)=\sum_{m=0}^{N-1} x(n+m)w(m)e^{-j\omega m}, \tag{28}$$

can be used instead of the standard FT. The adaptive STFT can be calculated with the same algorithm as in the previous subsection, for each time-instant. For an instant n, the algorithm can be performed by using samples $x(n+m)$, $m\in[0,N)$, where N is a width of the widest window from the considered set $\mathbf{N}$. Resulting adaptive FT can be further used for calculation of distributions from the Cohen class according to [15], [16], [17].

IV. Numerical Examples

Example 1. Consider a signal:

$$f(t)=\exp(j256\pi t)$$
$$+\frac{1}{2}\exp(j512\pi t)+\frac{1}{4}\exp(j768\pi t), \tag{29}$$

sampled with $\Delta t=1/1024$, within $t\in[-1,1]$. Signal is corrupted by

$$\nu(n)=\frac{\sigma}{\sqrt{2}}(\nu_1(n)+j\nu_2(n)), \tag{30}$$

where $\nu_i(n)$, $i=1,2$ are mutually independent white Gaussian noises with unitary variance. The adaptive algorithm is applied on the set of the Hanning windows whose widths are:

$$\mathbf{N}=[2048\ \ 1024\ 512\ 256\ 128\ 64\ 32\ 16\ 8\ 4]. \tag{31}$$

Results obtained with the proposed algorithm for $(\kappa+\Delta\kappa)=1$ are summarized in Fig. 1. The first column represents a small noise case $\sigma=0.1$, while the second and the third columns represent higher noise amounts, $\sigma=1$ and $\sigma=2$. The logarithm of the MSE, as a

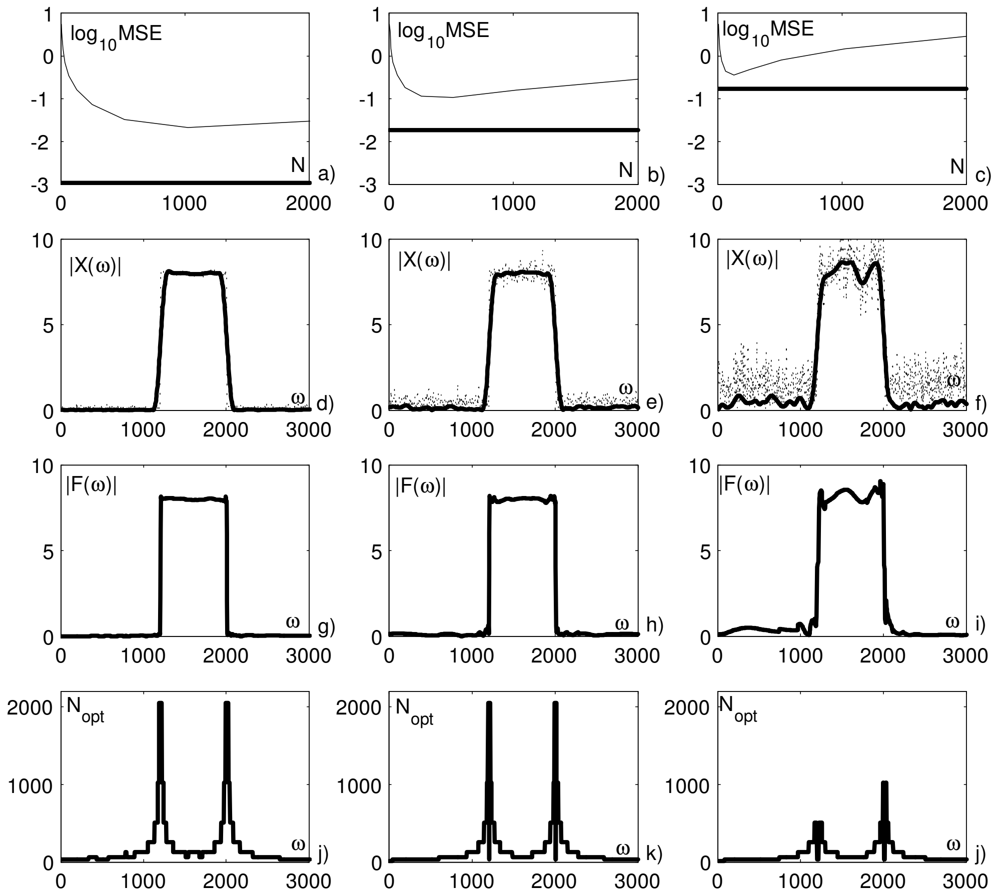

Fig. 5. Adaptive FT of signal whose FT represents an ideal pass-band filter: First column - $SNR = 20$dB; second column - $SNR = 10$dB; third column - $SNR = 0$dB. First row - logarithm of the MSE as function of the window width, thick line represents MSE for the adaptive algorithm; second row - FTs with the constant window width: $N = 2048$ - dashed line; $N = 128$ - thick line; third row - adaptive FT; fourth row - adaptive window width.

function of the window width, is shown in the first row of the Fig. 1. Thick lines represent the MSE obtained by the adaptive algorithm. By increasing the amount of noise, the MSE rapidly increases for the FT with wider windows, while for the FT with narrower windows it has almost a constant value. The FT obtained with windows of the constant width is shown in Fig. 1, second row. The FT with the widest window from the set **N**, $N = 2048$, is marked with dashed lines, while the FT with window $N = 128$ is marked with thick lines. The adaptive FT is shown in the third row of Fig. 1, while the adaptive window width is shown in Figure 1, forth row. It can be seen that by increasing the noise amount, window width around the signal component decreases.

Statistical analysis is performed on the presented signal, as well. The standard deviation range $\sigma \in [0, 3]$ has been considered. Logarithm of the MSE for three constant windows, from the set **N**, and for the adaptive algorithm, by using $(\kappa + \Delta\kappa) = (1, 1.25, 1.5)$, is shown in Fig. 2. Also, the adaptive algorithm with varying $(\kappa + \Delta\kappa)$ is calculated. The value $(\kappa + \Delta\kappa) = 1.75$ is used for $\omega = 0$. Value of $(\kappa + \Delta\kappa)$ is linearly decreased toward the maximal frequency, where $(\kappa + \Delta\kappa) = 1$ is used. Performance of the adaptive algorithm with $(\kappa + \Delta\kappa) = (1, 1.25, 1.5)$, and with a frequency

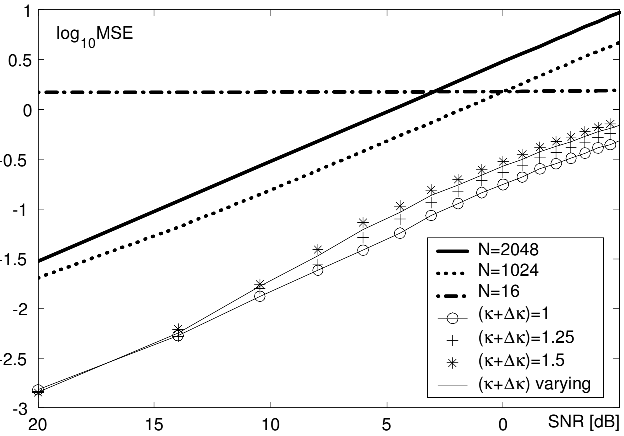

Fig. 6. MSE for various constant window widths and for adaptive algorithm with various $(\kappa + \Delta\kappa)$ for the case of the ideal pass-band filter.

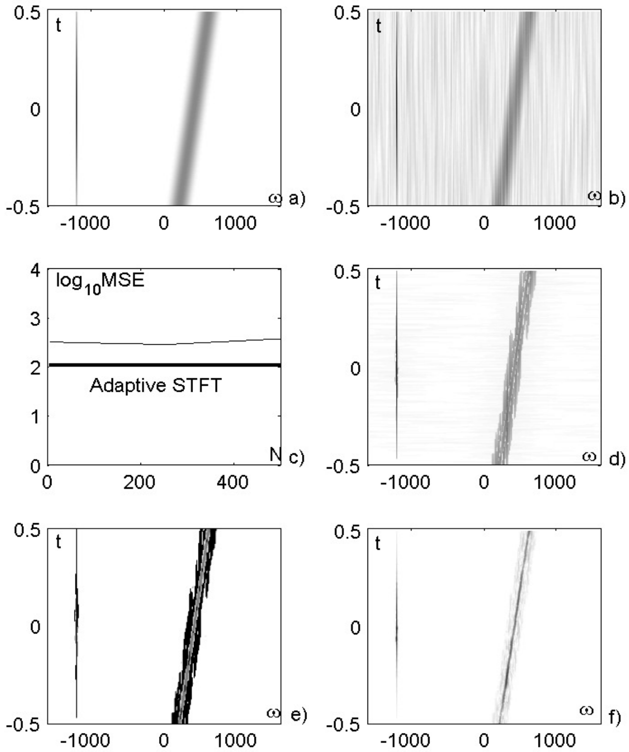

Fig. 7. Adaptive TF representation: a) STFT of the non-noisy signal calculated with the widest window; b) STFT of noisy signal; c) MSE as a function of the window width; d) adaptive STFT; e) adaptive window width; f) S-method calculated by using the adaptive STFT.

varying $(\kappa+\Delta\kappa)$ are almost the same. It shows that all values of $(\kappa+\Delta\kappa)$ from the proposed range produce similar performance. The adaptive algorithms perform better than the FT with any constant window from the considered set.

Example 2. Consider a sum of the complex sinusoid, damped complex sinusoid, and linear FM signal:

$$f(t) = 2\exp(j256\pi t)\exp(-32t^2)+$$
$$\exp(j512\pi t) + 2\exp(j64\pi t^2 + j768\pi t). \quad (32)$$

The numerical tests and the statistical analysis are performed under the same assumptions, as in the previous example. Results are given in Figs. 3 and 4.

Example 3. Consider a signal:

$$f(t) = 8\exp(j512\pi t)\frac{\sin(128\pi t)}{\pi t}, \quad (33)$$

that represents an ideal passband filter:

$$F(\omega) = \begin{cases} 8 & \omega \in [384\pi, 640\pi], \\ 0 & \text{elsewhere.} \end{cases} \quad (34)$$

Signal is sampled with $\Delta t = 1/1024$. Application of the proposed algorithm for signal (33), embedded in (30) with SNR equal to 20dB, 10dB and 0dB is shown in the columns of Fig. 5, respectively. Algorithm parameters are the same as in Example 1. It can be seen from the first two columns that a wide adaptive window is taken only around the cut-off frequencies, where $|F''(\omega)|$ is different from zero. This is in a full agreement with the proposed theory. However, outside this region the adaptive algorithm took narrow window since $|F''(\omega)| \approx 0$. In a heavier noise environment, third column in Fig. 5, adaptive window length around the cut-off frequencies is decreased in order to avoid noise influence. Statistical analysis for this case is shown in Fig. 6.

Example 4. TF analysis is performed on the signal

$$f(t) = 2\exp(j256\pi t)\exp(-32t^2)+$$
$$\exp(-j384\pi t)\exp(-8t^2), \quad (35)$$

corrupted by a white complex Gaussian noise (30) with $\sigma = 1.5$. The STFT of non-noisy signal (35), produced with the widest window from the considered set $N = 512$, is shown in Fig. 7a, while for the noisy signal it is shown in Fig. 7b. The adaptive algorithm with $(\kappa+\Delta\kappa) = 1.25$ is applied. The MSE for windows with constant widths, as well as for the adaptive algorithm, is shown in Fig. 7c. The adaptive STFT is shown in Fig. 7d, while the adaptive window width is shown in Fig. 7e. The S-method, as a distribution from the Cohen class that can be realized by using the adaptive STFT [16]

$$SM(n,\omega) = \sum_{l=-L}^{L} STFT(n,\omega + l\Delta\omega)\times$$
$$STFT^*(n,\omega - l\Delta\omega), \quad (36)$$

where $\Delta\omega$ is the frequency step, is shown in Fig. 7f. The value of $L = 3$ is used.

V. Conclusion

Non-parametric algorithm for determination of the adaptive FT is presented. The algorithm produces adaptive window length that gives a bias-to-variance trade-off, with the MSE smaller than for any constant window from the considered set. Generalization of the algorithm to the TF analysis of the signal with varying spectral content is done. The frequency-varying algorithm parameter $(\kappa+\Delta\kappa)$ is introduced in order to keep weak signal components that can be very important for audio quality of the speech signals. The algorithm produces almost the same MSE for a wide range of $(\kappa+\Delta\kappa)$ values.

VI. Acknowledgment

The work of I. Djurović is supported by the Postdoctoral Fellowship for Foreign Researchers of Japan Society for the Promotion of Science and the Ministry of Education, Culture, Sports, Science and Technology under Grant 01215. The work of LJ. Stanković is supported by the Volkswagen Stiftung, Federal Republic of Germany.

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