Systemic Approach in Aerial Target Tracking: Hardware and Software Implementation

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Abstract—**For obvious reasons of cost, discretion and reliability costs, locating and tracking aerial targets under an electromagnetically completely passive paradigm, relying exclusively on illuminators of opportunity, is very appealing for military but also civilian tasks. Such a passive radar system could exploit signals emitted by existing commercial television or radio stations or even satellite signals, such as the ones belonging to the GPS. The paper considers target locating and tracking using a network of passive receivers and/or non-cooperative illuminators (a multi-static radar configuration) by making use of the Doppler shift only. A novel concept, the systemic approach, is used to combine and interpret information available from different sensors. Both the formalization of the problem and the hardware and software implementation are presented. Implementation makes use of multi-component polynomial phase signal models and genetic algorithms. For increased performance, implementation on FPGA is envisaged. The paper ends with conclusions and perspectives.**

I. Introduction

Different methods [4] [5] [7] [11] can be used to perform target tracking in a passive radar system: TDOA (*Time Difference Of Arrival*), DOA (*Direction Of Arrival*), Doppler shift measurement, etc. TDOA method uses the time difference between the reflected signal's arrival and the direct signal's arrival, while DOA method examines the change in angle as a function of time. At least two stationary receivers are needed for targets to be unambiguously tracked.

The paper considers tracking using Doppler shift only, both for the challenge this problem presents and for its possible use as an alternative or complementary method for TDOA and DOA. Both single and multi-target scenarios are considered. This is similar to [10] and serves as a testbed for the concepts introduced in the remainder of the paper.

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A passive radar configuration, where solely the receiver is controlled and an illuminator of opportunity is used, is considered. Such a passive radar system could exploit signals emitted by existing commercial television – for example the Digital Video Broadcast – Terrestrial (DVB-T) transmitters – or radio stations or even satellite signals, such as the ones belonging to the GPS. This configuration is generally bi-static (or multi-static, if multiple receivers and illuminators are considered).

Let us assume a horizontal motion for the aerial target and consider the bi-static configuration depicted in Figure 1.

The Doppler shift (between illuminating and received signals) can be expressed as:

$$F_d = -\frac{f}{c}\dot{d} = -\frac{1}{\lambda}\dot{d} \tag{1}$$

where c is the velocity of light, f is the instantaneous frequency of the illuminating signal, $\lambda = c/f$ stands for its (instantaneous) wavelength and $\dot{d}$ is the derivative of the total length of the electromagnetic path:

$$\dot{d} = \underbrace{\frac{(x-x_i)v_x + (y-y_i)v_y}{\sqrt{(x-x_i)^2 + (y-y_i)^2}}}_{\dot{a}} + \underbrace{\frac{(x-x_r)v_x + (y-y_r)v_y}{\sqrt{(x-x_r)^2 + (y-y_r)^2}}}_{\dot{b}} \tag{2}$$

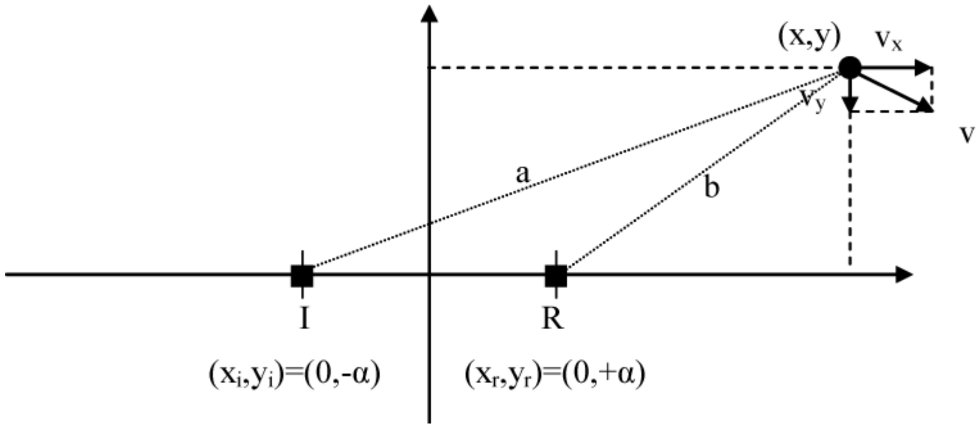

Fig. 1. Bi-static configuration.

Here, $v_x = \dot{x}$ and $v_y = \dot{y}$ are the components of target's (instantaneous) speed and, as such, functions of time.

The objective is to unambiguously locate targets using Doppler data alone, using a single transmitter-receiver pair, in addition to developing and comparing different multi-target tracking and association schemes and techniques.

Obviously, the Doppler shift given by (1) carries information about target motion and this is exploitable for target tracking. Note that less accuracy in Doppler shift estimation (spectral resolution) translates in less accuracy for the derived quantities (e.g. target location). It is apparent from (2) that such errors increase with the distance, as the latter links with the denominators in (2) that multiply the Doppler shift error.

To simplify the problem at hand, a number of assumptions will be made. Thus, non-manoeuvring targets, whose speed vector remains unchanged, are considered.

The remaining of the paper is organized as follows: Section 2 presents the issues arising when target locations is determined solely using the Doppler shift, as well as the solution of this problem, in a passive context (use of at least two receivers). Section 3 considers the general framework (multiple targets and multiple receivers) and a sequential method for discriminating targets. In Section 4 a new paradigm, the systemic approach, is introduced and exemplified through a global optimization problem formulation, solved using evolutionary algorithms. Since the Doppler-based tracking requires extracting the Doppler shift (frequency modulation), the WHAF method is presented in Section 5 tot this end. Section 6 illustrated the application of the systemic approach in tracking a pair of targets using several receivers, in an operationally realistic context. Finally, Section 7 exposes conclusions and perspectives of the work and acknowledgments are presented in Section 8.

II. ISSUES IN DOPPLER-BASED TRACKING

An elementary situation, where only one target and one receiver are involved, is used to illustrate the method, while exhibiting the main difficulty, i.e. the ambiguity problem.

The notations:

$$u = \begin{pmatrix} \frac{x-x_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} + \frac{x-x_r}{\sqrt{(x-x_r)^2+(y-y_r)^2}} \\ \frac{y-y_i}{\sqrt{(x-x_i)^2+(y-y_i)^2}} + \frac{y-y_r}{\sqrt{(x-x_r)^2+(y-y_r)^2}} \end{pmatrix} \tag{3}$$

$$v = \begin{pmatrix} v_x \\ v_y \end{pmatrix} \tag{4}$$

allow rewriting the Doppler shift as:

$$F_d = -\frac{f}{c} < u, v >= -\frac{f}{c} u^T v =$$

$$= -\frac{f}{c} \left(u_x v_x + u_y v_y \right) \tag{5}$$

where u_x and u_y are the components of u vector.

The inner product in (5) is symmetrical and the same Doppler shift is produced by any of the following pairs:

$$u_x v_x + u_y v_y = \begin{pmatrix} u_x \\ u_y \end{pmatrix}^T \begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} -u_x \\ u_y \end{pmatrix}^T \begin{pmatrix} -v_x \\ v_y \end{pmatrix} = \begin{pmatrix} u_x \\ -u_y \end{pmatrix}^T \begin{pmatrix} v_x \\ -v_y \end{pmatrix} = \begin{pmatrix} -u_x \\ -u_y \end{pmatrix}^T \begin{pmatrix} -v_x \\ -v_y \end{pmatrix} \quad (6)$$

This results in 4 ambiguous (or symmetric) target states (positions and speeds). The tracking based on the solely Doppler shift is ambiguous. Synthetically, these targets are expressed as the state vectors below:

$$X_1 = \begin{bmatrix} x_0 & v_x & y_0 & v_y \end{bmatrix}^T \quad (7)$$

$$X_2 = \begin{bmatrix} x_0 & v_x & -y_0 & -v_y \end{bmatrix}^T \quad (8)$$

$$X_3 = \begin{bmatrix} -x_0 & -v_x & y_0 & v_y \end{bmatrix}^T \quad (9)$$

$$X_4 = \begin{bmatrix} -x_0 & -v_x & -y_0 & -v_y \end{bmatrix}^T \quad (10)$$

To each measured Doppler shift corresponding to a given target (assume X_1), 3 other target states (X_2, X_3 and X_4) will appear as spurious solutions. Thus, locating and tracking are ambiguous (of order 4). All 4 ambiguous target states are depicted in Figure 2.

Since 4 distinct states (trajectories) induce identical Doppler shifts, the initial target state cannot be uniquely determined from the Doppler shift alone. This problem may be dealt with by considering an additional receiver (or, alternatively, a transmitter).

Now, let us illustrate the usefulness of adding a second receiver. Recall that a single receiver will not allow pointing out which one of the 4 targets in Figure 2 is real.

A second receiver (R_2 in Figure 2) is used to break the ambiguity. The recorded Doppler shift at R_1 gives 4 ambiguous targets. For each one, the would-be-measured Doppler shift at R_2 is computed. Finally, cost of each possible target is computed:

$$C_i = \sum_{k=0}^{N-1} \left\| m_2[k] - h_{2i}[k] \right\|^2 \quad (11)$$

where $m_2[k]$ is the actual measured Doppler shift at R_2, $h_{2i}[k]$ is the estimated Doppler shift at R_2 produced by the i^{th} symmetric solution ($i = \overline{1,4}$) and N is the number of samples. The solution that gives the smallest of the four costs represents the actual target state.

In Figure 3, Doppler shifts measured at the level of R_1 and R_2 are shown for all the 4 ambiguous solutions.

As expected, the Doppler shifts superpose perfectly at receiver R_1, thus not allowing for separation. On the other hand, just one of the four Doppler shifts matches the measured Doppler shift at receiver R_2 (the white dots).

This simple example shows how is possible to break the ambiguity by adding a second receiver. This approach can be further generalized for single and multiple target tracking.

III. General Framework for Target Tracking

The exposed approach is extendable to the general case, where a number of N targets are tracked using M receivers (a single transmitter is required). We denote by $(m_{ij})_{\substack{i=\overline{1,M} \\ j=\overline{1,N}}}$ the Doppler shift associated to the target j, as recorded by the receiver i (target numbering is arbitrary at each receiver, as there is no clue to associate them). The vector $m_{i,j}$ stands for the set of Doppler shift values recorded at K discrete measurement times, i.e. $m_{i,j} = \begin{bmatrix} m_{ij}[0] & m_{ij}[1] & ... & m_{ij}[K-1] \end{bmatrix}$.

The goal is to associate targets and measured Doppler shifts through a bijective function. The procedure involves iteratively blocking out the target having the lowest cost and its associated Doppler shifts.

First, the Doppler shifts measured by R_1 are arbitrarily numbered, the Doppler shift m_{1j} being associated to target $j = \overline{1,N}$.

Second, for each vector m_{1j}, the set of 4 ambiguous states are computed. Generally, only

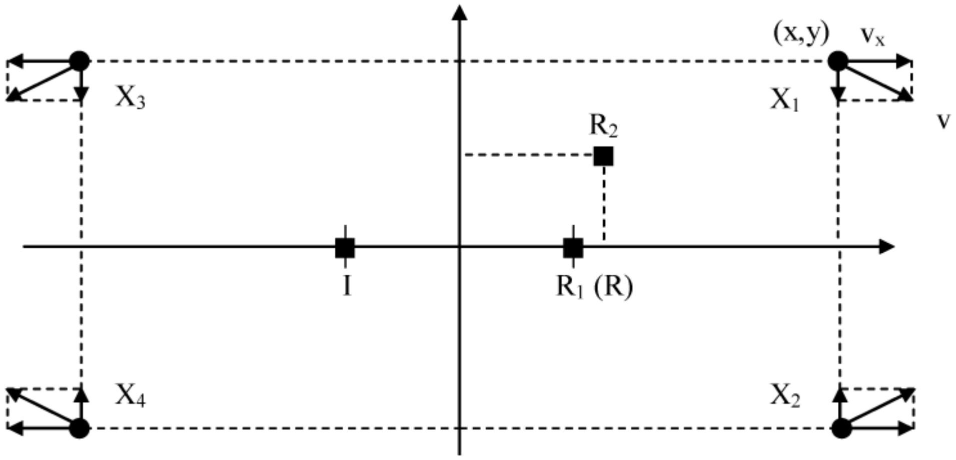

Fig. 2. Target tracking ambiguity.

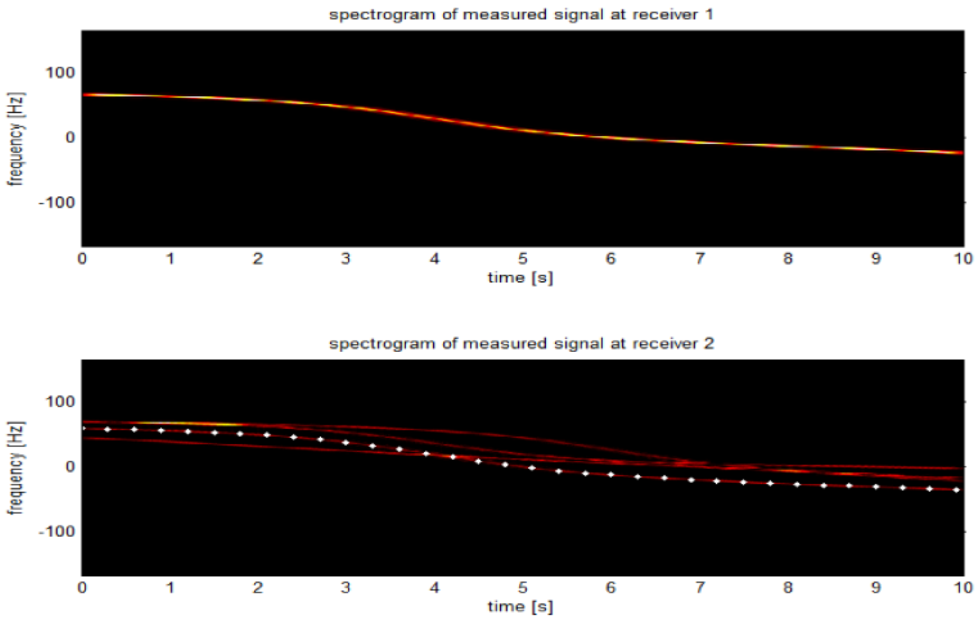

Fig. 3. Measured and estimated Doppler shifts at R_1 (R) and R_2 for the four ambiguous solutions.

one of these is a real target. A number of $4N$ targets will be then examined.

Third, $4N$ Doppler shifts, as it would have been measured at R_2 for the considered $4N$ targets, are computed.

Fourth, the cost functions for each of the $4N$ solutions are computed, with respect to the R_2 receiver. Note that the $4N$ possible solutions have to be compared against the N Doppler shifts that R_2 actually records. So, a number of $4N^2$ costs will be computed, one for each combination.

The minimum cost will identify the real target, so that it may be dropped from the original set of N targets to be tracked. The corresponding 4 Doppler shifts (real and spurious) will be dropped, too.

Then the procedure is restarted, this time to analyze the remaining $4N^2 - 4N$ costs only, until the correct identification of all targets.

While relatively simple and fast, the technique uses only a pair of receivers. More, implementing it may become complicated if information is missing (e.g. if one ore more Doppler shifts are noisy or simply unavailable at R_2). Finally, there are 2 sequences in which the information from R_1 and R_2 may be examined (first R_1, then R_2 or first R_2, then R_1). For increased robustness, averaging their results is an option.

One way to embed information from more sensors is to consider different pairs of receivers (R_i and R_j). Coordinates of targets must be recomputed each time, with reference to the new coordinate system, whose Ox axis is linked with R_i and R_j.

Overall, the pairwise estimation algorithm should be applied 2^M times, each time a number of $4N^2$ costs being analyzed. A final average step will conclude the method.

IV. Towards a systemic approach and global optimization techniques

A systemic approach is a methodology than first considers the global perspective of the problem at hand [15]. Unlike analytic methods, which follow an iterative algorithm and use a single starting point in searching the solution, under the systemic approach paradigm, the problem is considered as a whole. Function of the specific problem, the latter could specifically mean: considering multiple (even an infinity) of starting points who are simultaneously improved, integrating the whole available information in each processing step, combining information provided by a number of analytic methods, etc.

This section presents a way for globally and simultaneously assessing information provided by all receivers.

The same configuration (N targets and M receivers) as in the previous section is considered. All possible combinations of Doppler shifts, for each receiver, are formed. This gives $(N!)^{M-1}$ combinations. Each combination s is a permutation of the lower $M-1$ lines of the vector matrix $(m_{ij})_{\substack{i=\overline{1,M}\\ j=\overline{1,N}}}$ (the first line remains unchanged). For each combination, each column of the correspondingly permutated vector matrix is seen as the response of a possible real target. Each of these combinations are compared against the actual measured Doppler shifts $(h_{ij})_{\substack{i=\overline{1,M}\\ j=\overline{1,N}}}$ where the numbering of intercepted Doppler shifts at each receiver i, while remaining fixed for the rest of the method, is arbitrary.

The question is to determine, among all combinations, the true solution of the problem, i.e. the combination exhibiting the minimum cost. The task is thus reformulated as an optimization problem.

For each combination, the global cost of Doppler shifts is computed. Then, the global optimization solution is given by:

$$sol = \underset{comb \in \{set\,of\,combinations\}}{\arg\min} C \tag{12}$$

where $C = \sum_{i=1}^{M} C_i$ is the sum of all partial costs C_i.

The partial cost C_i reflects the information provided by sole the receiver i and is given by:

$$C_i = \sum_{j=1}^{N} \|s_{i,j} - h_{i,j}\| \tag{13}$$

In the equation above, $h_{i,j}$ are the Doppler shifts as recorded by the receiver i, $i = \overline{1,M}$. Again, the vector notation is a substitute for the temporal samples of the measured Doppler shift (its values are instantaneous frequencies, not signal samples), so that:

$$h_{i,j} = [\ h_i(X_j)[0] \quad h_i(X_j)[1] \quad \ldots \quad h_i(X_j)[K-1]] \tag{14}$$

where K is the total number of samples and X_j is the state (vector of motion parameters) of the target j.

Minimizing C is not a trivial task. A large number of variables are involved and the cost function could exhibit a lot of local minima. One may be constrained to make use of specific optimization methods, such as genetic algorithms [6], to solve this task.

The Genetic Algorithms (GA) and Evolutionary Strategies (ES) [3] [16] aim at optimizing a multi-objective problem by minimizing a cost function whose global minimum corresponds to the solution of the problem. The main asset of GA is the capability to cover the problem with an initial population of candidates who evolve from generation to generation towards the most adapted candidate, imitating natural evolution. Unlike ES who directly modify raw parameters in the process, a GA codes the parameters of its candidates through some kind of representation. The former also starts from a unique candidate that will try to find his way to the global minimum of the cost function, modifying its parameters according to an adopted pattern search strategy. Here we attempt to use advantages of both methods to solve the Doppler based aerial target tracking, combining the two strategies into a hybrid (and systemic-paradigm) algorithm.

To deal with the problem of target tracking, a population of individuals is formed, where each member encodes a possible global solution (positions and speeds of all targets):

$$X_m = \Big[(x_i)_{i=\overline{1,N}}, (y_i)_{i=\overline{1,N}},$$

$$(V_{x_i})_{i=\overline{1,N}}, (V_{y_i})_{i=\overline{1,N}}\Big] \quad (15)$$

where x_i, y_i give the position of the target i, and V_{x_i}, V_{y_i} the speed (vertical components are neglected).

The initial population $[X_1, X_2, ..., X_m]$ composed of P members will iteratively evolve, the individuals hopefully being directed towards the global minimum of the cost function in (12).

The algorithm thus performs optimization in a $4N$ dimension space, minimizing gaps between the actual Doppler shift and the ones created by the candidates.

Implementation use selection of best individuals with a random component (roulette), random mutations and random crossovers between best performing members.

Unlike GA, no coding (of parameters into genetic representation) is performed. Instead, the raw parameters are directly modified, the way evolutionary strategies would do.

V. Extracting the Doppler shifts. Warped high-order ambiguity function

To extract the Doppler shifts from the signals measured at receivers, the Warped High-order Ambiguity Function (WHAF) [9] is used. This is an enhanced version of the Product High-order Ambiguity Function (PHAF) [2], the latter being born from the High-order Ambiguity Function (HAF) [12].

The HAF-based method approaches signals' phase in a polynomial way, within a finite interval of time. However, as it was illustrated in [13], [17], the algorithm presents some limitations, related to the *noise robustness* and the *cross-terms presence*. These issues are considered critical with respect to the considered application, because of the possible jamming and the inherent complexity of signals. In order to solve these aspects, the concepts of *multi-lag HAF* (mlHAF) [17] and Product HAF (PHAF) |2] have been proposed. The mlHAF is the Fourier transform of the generalization of the high-order instantaneous moment HIM:

$$HIM_K\left[s(t);\boldsymbol{\tau_{K-1}}\right] =$$
$$= HIM_{K-1}\left[s(t+\tau_{K-1});\boldsymbol{\tau_{K-2}}\right]$$
$$\times HIM^{*}_{K-1}\left[s(t-\tau_{K-1});\boldsymbol{\tau_{K-2}}\right]$$

$$mlHAF_K\left[s;\alpha,\boldsymbol{\tau}\right] =$$
$$= \int_{-\infty}^{\infty} HIM_K\left[s(t);\boldsymbol{\tau}\right] e^{-j\alpha t} dt \quad (16)$$

where $\boldsymbol{\tau_i} = (\boldsymbol{\tau_1}, \boldsymbol{\tau_2}, ..., \boldsymbol{\tau_i})$ is the lag set, while the PHAF is the product of several mlHAFs computed for different lag sets:

$$PHAF(\alpha;T) =$$
$$= \prod_{l=1}^{L} mlHAF_K\left[s;\frac{\prod_{i=1}^{K-1}\tau_i^{(l)}}{\prod_{i=1}^{K-1}\tau_i^{(1)}}\alpha, \tau_{K-1}^{(l)}\right],$$
$$T = \left\{\tau_{K-1}^{(l)}\right\}_{l=\overline{1,L}} \quad (17)$$
$$\tau_{K-1}^{(l)} = \{\tau_i\}_{i=\overline{1,K-1}}$$

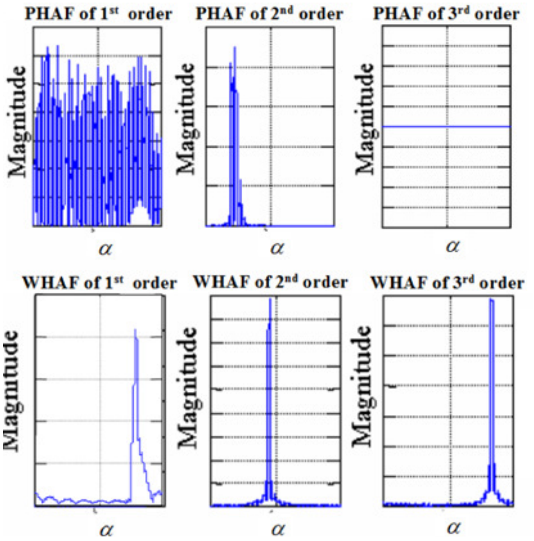

Fig. 4. Example of an abnormal result related to product

The polynomial-phase model used for the analyzed signal is:

$$s\left(t\right)=Ae^{j\phi(t)}=Ae^{\left[j\sum_{k=0}^{K}a_{k}t^{k}\right]}. \qquad (18)$$

However, proper use of PHAF is conditioned by the selection of good lag sets, often determined by empirical trials. This constitutes a considerable limitation in practical applications, since multiplying mlHAFs could lead to abnormal situation if lags are not appropriate (notably, wrong estimates for the polynomial coefficients). A new method – the Warped HAF (WHAF) – relies on the *axis transformation* (or axis warping) concept [1] to this problem. It gives also a general way for the lag set selection. Details are given in [8].

Consider, for example, the results depicted in Figure 4 for the following signal:

$$s\left(t\right)=e^{\left(j2\pi\left(0.25t-4.55\cdot 10^{-4}t^{2}+1.78\cdot 10^{-6}t^{3}\right)\right)} \qquad (19)$$

Note that, unlike the 3^{rd}order PHAF (Figure 4 upper right), the 3^{rd}order WHAF (Figure 4 lower right) provides an easily identifiable maximum.

Assuming a polynomial phase model for the signal (and knowing the dependence law between α and τ, namely $\alpha_k\left(\tau\right)=k!\tau^{k-1}a_k$), the WHAF method uses the following set of lags:

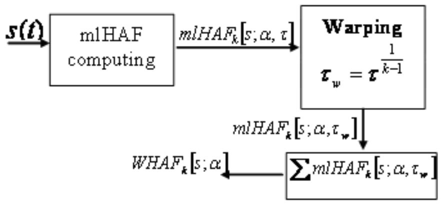

Fig. 5. WHAF block diagram

$$\tau_w=\tau^{\frac{1}{k-1}} \qquad (20)$$

The effect of the warping function (20) in the frequency-lag plane consists of disposing the mlHAFs peaks on parallel lines with the lag axis. Therefore, an idea to exploit this property is to sum the mlHAFs obtained for the warped lag set. We generate the WHAF as:

$$WHAF_k[s;\alpha]=\sum mlHAF_k\left[s;\alpha,\tau_w\right] \qquad (21)$$

which peaks at the locations $\alpha_k=k!a_k$. The term $mlHAF_k\left[s;\alpha,\tau_w\right]$ represents the multi-lag HAF obtained using the warping operator given in (20).

The estimation of the polynomial coefficients, in the case of multi-lag HAF-based methods (PHAF or WHAF), the M^{th} order polynomial coefficient is obtained with the following estimator:

$$\hat{a}_M=\frac{1}{2^{M-1}M!\prod_{k=1}^{M-1}\tau_k}\times\arg\max_{\alpha}\left|WHAF_M\left[s,\alpha\right]\right| \qquad (22)$$

where s is the analyzed signal and $\{\tau_k\}$ is the lag set used for WHAF computation. As indicated by this expression, the estimation of the M^{th} order polynomial coefficient is dependent on the estimation of the frequency location corresponding to the WHAF maxima:$f_0=\arg\max_{\alpha}\left|WHAF_M\left[s,\alpha\right]\right|$. Using a first order approximation, the measured value of this frequency is affected by a perturbation term: $\hat{f}_0=f_0+\delta f$. Under this assumption, it may be shown [2] [14] that the bias of the estimator [14] is zero, so the mlHAF/WHAF-based estimators for the polynomial-phase coefficients are unbiased.

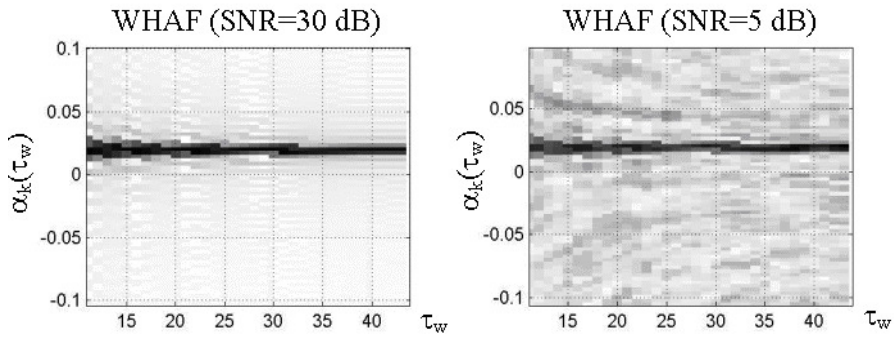

Fig. 6. Warping High-Order Ambiguity Function

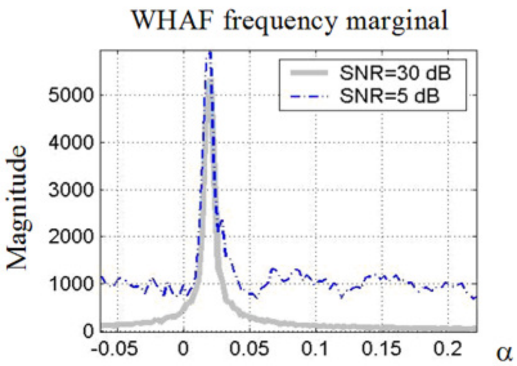

Fig. 7. Localization of WHAF maxima

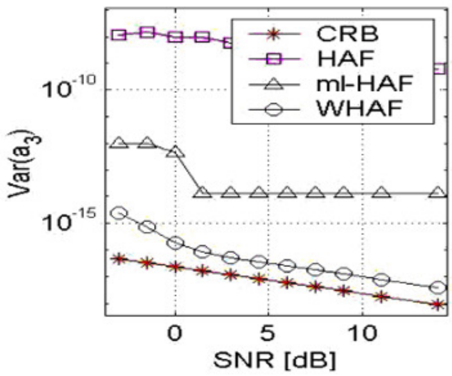

Fig. 8. Variance of the estimation vs. SNR

The WHAF block diagram is given in Figure 5.

The WHAF for the signal in (19) is plotted in the Figure 6. The noised version of this signal (SNR=5 dB) is also examined.

Note that the peaks of mlHAF, computed for an arbitrary lag set, are arranged around the same (horizontal) line (the $k!a_k$ line), thus facilitating the estimation of k^{th} order polynomial coefficient. This is illustrated in the Figure 7 where the frequency marginals of WHAF values, for both SNRs, are plotted.

As shown on Figure 7, in spite of noise presence, the frequency coordinate associated to 3^{rd} order polynomial coefficient is the same as in the noise reduced case (SNR=30 dB).

It may be shown that WHAF-based estimation method provides better performances compared to the mlHAF procedure. The variances of the estimators are close to the Cramer-Rao Bound [13] [8] (see Figure 8).

More, WHAF method is operationally more advantageous than the PHAF: since the lag sets are object of warping operation, their choice does not influence the result.

The next example (Figure 9) illustrates the capability of the WHAF approach to deal with noisy multi-component signals. We consider a two-component signal given by:

$$x(t) = e^{\left(j2\pi\left(0.25t+9.7656\cdot 10^{-4}t^2+4.2915\cdot 10^{-6}t^3\right)\right)}$$

$$+e^{\left(j2\pi\left(0.15t+4.8828\cdot 10^{-4}t^2+1.9073\cdot 10^{-6}t^3\right)\right)} + w(t) \tag{23}$$

where w is a white Gaussian noise (SNR=8 dB).

VI. Sample case and results

Sample case considers:

-3 receivers located nearly 10 km away from each others;

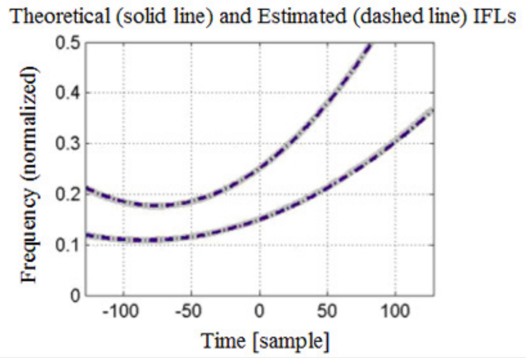

Fig. 9. WHAF approach for a two component signal

-1 illuminator of opportunity, located at 50 km from the receivers, transmitting at 300 MHz;

-2 targets whose speeds are below MACH 1.

The analysis tempts to use a realistic approach, in which information is gathered as it becomes available and when the underlying (genetic) algorithm has the possibility to use it in order to update its estimation. This is achieved by running and updating the algorithm in time slices of 3 seconds, although the length of the time slice is somehow arbitrarily picked (based on the computation time obtained on an ordinary PC equipped with Matlab). Relying on faster hardware, this value may be reduced, for accuracy reasons or because of other constraints. Indeed, a faster pace of the iterative estimating procedure will result in increased accuracy while, on the other hand, for some kind of transmitters (e.g. the DVB-T transmitters), the time the target stays in the effective range of the considered illuminator of opportunity is generally less than a second.

The analysis steps are described below.

1st step: signals received on receivers are translated into a base-band of 20 Hz by undersampling (300MHz->20Hz), Doppler shifts being less than 20Hz.

2nd step: extracting the Doppler shifts by PHAF and estimating polynomial orders, assuming target motions can be considered linear and uniform for about 3 seconds. Doppler shifts values are thus extracted from each signal, and passed to the tracking algorithm.

3rd step: running the tracking algorithm for 3 seconds, approaching the global solution in an 8 dimension space (4 per target), looking for the targets in the covered area. Accuracy at this stage is around 10 kilometers for positions, 30m/s for speed

4th step: updating the Doppler shift: step 1, 2, and 3 are repeated. Accuracy at this stage is around 500m for positions, 5m/s for speed.

5th step: repeating step 4. Accuracy at this stage is around 10m for positions, 1m/s for speed.

After two updates, the tracking algorithm has been able to precisely locate both targets, in real-time.

In Figures 12 and 13:

1. (blue line) Doppler shifts are extracted at times 0.5, 3.5 and 6.5, then a linear model of the targets are provided for 3 seconds;
2. (green line) The estimation, getting more and more accurate with time.

Main limitations of the proposed implementation are:

1. targets can maneuver but their speed is assumed relatively constant for 3 seconds;
2. only noise-free scenario is considered;
3. sudden variations of speed can fool the algorithm.

The 3 seconds interval above is the time needed for the tracking algorithm to be completed. It was implemented in MATLAB, so this accounts for the rather long latency. However, this can be improved, since the algorithm uses much parallel processing, and implementing it on specialized devices such as FPGA should drastically decrease the execution time.

VII. Conclusions and perspectives

The paper considers a limited case, in which targets are assumed to have relatively smooth motion, assumed to be linear for about 3 seconds, while their vertical speed is neglected. Obviously, this is not the most realistic scenario. Manoeuvring targets, like fighters, largely overcome the limits of these assumptions.

Moreover, noise should also be considered and the behaviour of systemic and sequential methods should be comparatively studied.

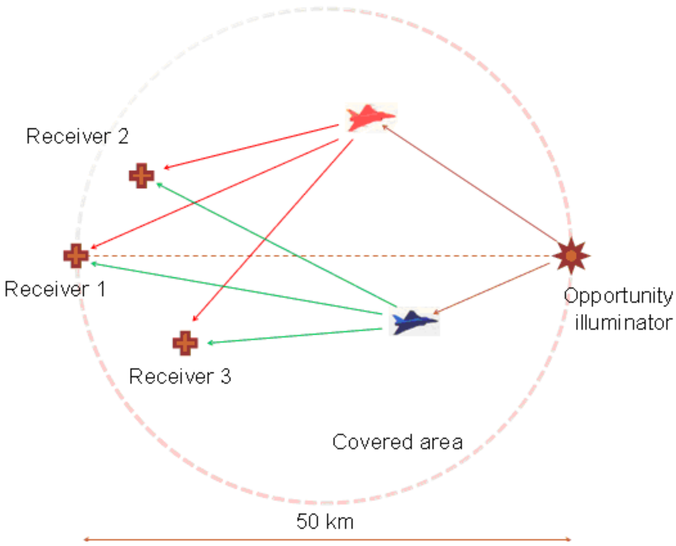

Fig. 10. Analyzed scenario

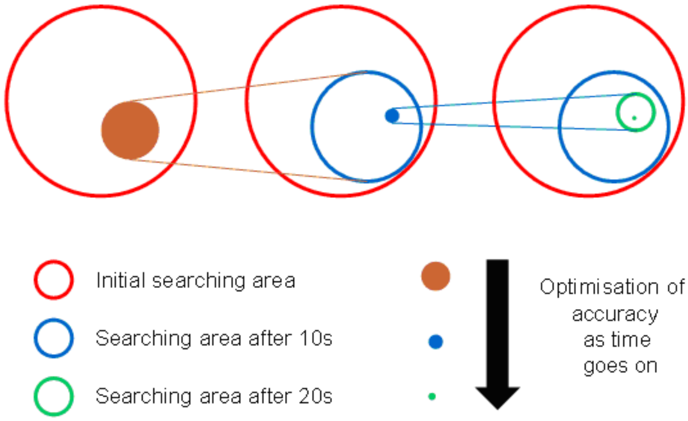

Fig. 11. Real time tracking

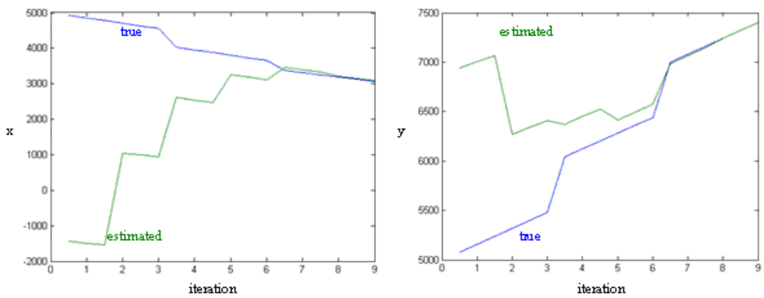

Fig. 12. Approaching x_1 and y_1 iteratively

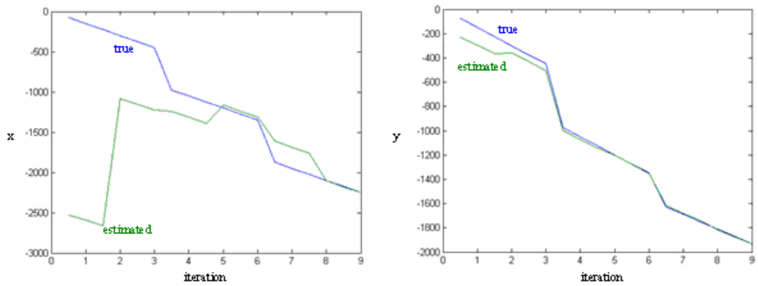

Fig. 13. Approaching x_2 and y_2 iteratively

This is currently under development and will make the subject of a future publication.

Increasing the tracking algorithm calculation speed would allow us first to decrease the 3 seconds linear assumption, which would enable more robust and accurate tracking, even for brutal motion. Then, we could consider estimating other motion parameters, at the expense of an increased computation cost.

For example, it would be possible to take into account accelerations of the target. In this case, the target could be described by an expanded state vector:

$$X_1 = [x_0, \xi_1, \xi_2, ..., \xi_G, y_0, \eta_1, ..., \eta_G]^T \quad (24)$$

where x_0 and y_0 are target coordinates at $t = 0$, while $\xi_1, ..., \xi_G$ and $\eta_1, ..., \eta_G$ are their derivatives (accelerations), while G is the degree of motion approximating polynomial:

$$x(t) = x_0 + \xi_1(t - t_0) + \frac{1}{2}\xi_2(t - t_0)^2 + ... \quad (25)$$

and

$$y(t) = y_0 + \eta_1(t - t_0) + \frac{1}{2}\eta_2(t - t_0)^2 + ..., \quad (26)$$

respectively.

These suggested improvements will turn the proposed method into a field deployable solution, thus confirming the possibilities open by the use of passive radars.

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