

Motion Parameters Estimation by Using Time-Frequency Representations

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Abstract— The estimation of motion parameters of moving objects by using variable μ -propagation and time-frequency representations is proposed. The spectrogram and the Wigner distribution, two basic time-frequency distributions, are used. Both the velocity and the initial position can be accurately estimated by this approach.

I. INTRODUCTION AND MOTION MODEL

The subspace-based line detection (SLIDE) algorithm for high resolution estimation of the line parameters, as well as its numerous interesting applications, has been proposed in [1]. Recently, the SLIDE algorithm has been used for estimation of velocity of moving objects in video-sequences [2]. Motion estimation is a very important topic in video-signal processing (video-signal compression, road tracking, aerial photograph processing, tomography, traffic control, meteorology, etc.). The velocity determination problem, by using the SLIDE algorithm and constant μ -propagation, is reduced to the frequency estimation in the Fourier domain.

In this letter, variable μ -propagation is introduced and velocity determination is performed based on the instantaneous frequency estimation. Time-frequency representations for the instantaneous frequency estimation are used [4], [5]. The variable μ -propagation provides estimation of initial position and velocity in a single step.

An image containing a moving object can be represented as $i(x, y, t) = f(x, y) + s(x - x_0 - v_x t, y - y_0 - v_y t)$ where $s(x, y)$ is moving object, $f(x, y)$ is contrast background, while t is the considered frame. The parameters of the moving object are: initial position (x_0, y_0) and velocity (v_x, v_y) . The estimation of the motion parameters of the moving object can be done

by using the image projections to the x and y axes. We will consider only the projection to the x -axis: $P(x, t) = \sum_y i(x, y, t)$, which will be used for determination of x_0 and v_x . The procedure which provides the estimation of y_0 and v_y is a straightforward extension.

The projection $P(x, t)$ can be written as:

$$P(x, t) = \sum_y f(x, y) + \sum_y s(x - x_0 - v_x t, y - y_0 - v_y t) = F(x) + S(x - x_0 - v_x t) \quad (1)$$

where $\sum_y s(x, y) = S(x)$ and $\sum_y f(x, y) = F(x)$.

In order to eliminate background influence, the derivative of projection with respect to t is used:

$$\begin{aligned} \partial P(x, t) / \partial t &= v_x \partial S(x - x_0 - v_x t) / \partial x = \\ &= \Pi(x - x_0 - v_x t) \simeq P(x, t + 1) - P(x, t). \end{aligned}$$

By using the SLIDE model, with the constant μ -propagation and $\partial P(x, t) / \partial t$, we form the signal:

$$z_{xx}(t) = \sum_x \Pi(x - x_0 - v_x t) e^{j\mu x} \quad (2)$$

whose frequency corresponds to the velocity of the moving object. The Fourier transform of the $z_{xx}(t)$ is: $FT\{z_{xx}(t)\} = 2\pi e^{jx_0\mu} \Phi(\mu) \delta(\omega_x - \mu v_x)$ where $\Phi(\mu)$ is the Fourier transform of the projection's function $\Pi(x)$. Note that, if $\Pi(x - x_0 - v_x t)$ is close to $\delta(x - x_0 - v_x t)$, the analysis is significantly simplified and we obtain: $FT\{z_{xx}(t)\} \simeq 2\pi \delta(\omega_x - \mu v_x)$. Thus the motion parameter v_x is determined as a position of the maximum of the Fourier transform scaled with the parameter μ .

II. MOTION ESTIMATION BY USING VARIABLE μ -PROPAGATION

We will introduce the variable μ -propagation in the form:

$$z_{xx^2}(t) = \sum_x \Pi(x - x_0 - v_x t) e^{j\mu x^2}. \quad (3)$$

Having in mind the previous considerations, it is obvious that $z_{xx^2}(t)$ has the form of a linear frequency modulated (FM) signal. For the sake of simplicity we will consider the case when $\Pi(x - x_0 - v_x t)$ is close to $\delta(x - x_0 - v_x t)$. In this case we obtain: $z_{xx^2}(t) = e^{j\mu(x_0 + v_x t)^2}$. For analysis of this type of signal time-frequency distributions are a more appropriate tool than the Fourier transform. The simplest time-frequency distribution is the spectrogram, given as the squared magnitude of the short-time Fourier transform:

$$\begin{aligned} SPEC_x(t, \omega) &= |STFT(t, \omega)|^2 = \\ &= \left| \sum_{\tau} z_{xx^2}(t + \tau) w(\tau) e^{-j\omega\tau} \right|^2 \end{aligned}$$

where $w(\tau)$ is the window function. The spectrogram of the linear FM signal can be approximately represented as [3]:

$$SPEC_x(t, \omega) \approx kw^2 \left(\frac{\omega - \mu x_0 v_x - \mu v_x^2 t}{\mu v_x^2} \right). \quad (4)$$

>From equation (4) it can be readily seen that the maxima of $SPEC_x(t, \omega)$ are along the line $\omega = \mu x_0 v_x + \mu v_x^2 t$. However, the spectrogram exhibits a low time-frequency resolution, as well as low concentration for this kind of signals. Since the Wigner distribution is an ideally concentrated distribution along the instantaneous frequency for linear FM signals, it is a more appropriate tool for this purpose [4]. The Wigner distribution is defined by:

$$\begin{aligned} WD_x(t, \omega) &= \\ &= \sum_{\tau} w(\tau) z_{xx^2}(t + \tau) z_{xx^2}^*(t - \tau) e^{-j2\omega\tau}. \end{aligned}$$

The Wigner distribution of the signal $z_{xx^2}(t) = e^{j\mu(x_0 + v_x t)^2}$ is:

$$WD_x(t, \omega) = 2\pi W(\omega - \mu x_0 v_x - \mu v_x^2 t) \simeq$$

TABLE I
ESTIMATED VELOCITIES.

	Exact	SPEC	WD
v_{x1}	2.3	2.3531	2.2913
v_{y1}	2.2	2.2935	2.1886
v_{x2}	1.5	1.2933	1.4939
v_{y2}	1.2	1.0023	1.2041
x_0	15	18.63	15.07
y_0	9	10.62	8.47

$$\simeq 2\pi\delta(\omega - \mu x_0 v_x - \mu v_x^2 t) = 2\pi\delta(\omega - b_x - a_x t).$$

where a wide window is assumed. We conclude that by using the variable μ -propagation, we perform mapping of the projections to the time-frequency chirp presentation. Thus, the parameters v_x and x_0 can be easily determined from the $WD_x(t, \omega)$. The parameters are: $\pm v_x = \sqrt{a_x/\mu}$ and $x_0 = b_x/\mu v_x$. On the base of projection derivative $\partial P(x, t)/\partial t$ the ambiguity in sign of velocity is avoided. Illustration of a parameter determination is shown in Fig.1a.

Example: The object is moving through the image with contrast background. The image size is 256×256 pixels, while the object size is 8×8 . The initial position of the motion object is $(x_0, y_0) = (15, 9)$, Fig.1b. For the first 50 frames velocity parameters are $(v_{x1}, v_{y1}) = (2.3, 2.2)$, while in the next 50 frames they are $(v_{x2}, v_{y2}) = (1.5, 1.2)$. The spectrogram and the Wigner distribution are shown on the Figs.2a and b. The instantaneous frequency estimation is shown in Fig.2c. The estimated velocities are given in the Table I. These results confirm that for signal with linear instantaneous frequency the Wigner distribution exhibits a more accurate instantaneous frequency estimation than the spectrogram. Here, it means more accurate motion parameters estimation.

III. CONCLUSION

In this letter an approach to motion estimation based on the variable μ -propagation and time-frequency representations is proposed. It is shown that the estimation is significantly more reliable by using the Wigner distribution instead of the short-time Fourier transform.

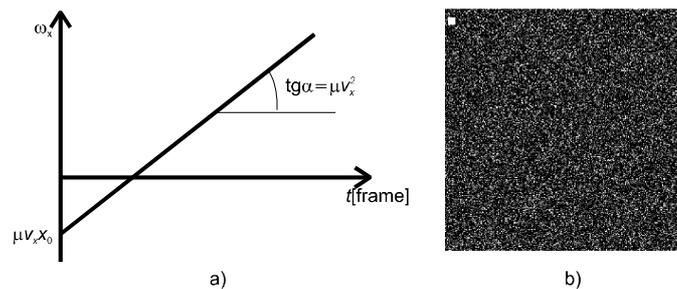


Fig. 1. Illustration of determination of motion parameters and initial frame.

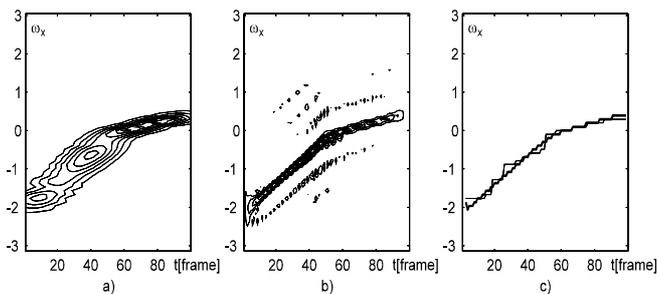


Fig. 2. Motion estimation for moving object with variable velocity: a) Spectrogram; b) Wigner distribution; c) Velocity estimation: a) Thin line - spectrogram; b) Thick line - Wigner distribution.

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