Multiple STFTs-based approach for chaos detection in oscillatory circuits

Igor Djurović and Vesna Rubežić

Abstract— **This paper deals with the recently proposed detector of chaotic states in nonlinear oscillatory circuits [1]. Detector is based on time-frequency representations of signals generated by oscillators. It can be realized by using cross-terms free (or reduced) representations such as the short-time Fourier transform (STFT). It has been noticed that its accuracy substantially depends on the applied window width used in the time-frequency representation. The subject of this paper is the accuracy analysis of such detector in the noisy environment with respect to the window width in the time-frequency representation. Based on this analysis a multiple STFTs-based approach has been proposed producing better results than its single window counterpart.**

I. Introduction

Nonlinear systems may exhibit chaotic behavior under specific conditions. Chaos has been noticed in electric circuits and systems, mechanical, optical and other systems, as well as in nature [2]-[5]. Detection and possible prediction of chaotic behavior have attracted significant attention of researchers for a long time. There are several techniques for chaos detection. The most frequently used technique is calculation of the Lyapunov's exponents [6]-[8]. Existence of at least one positive Lyapunov's exponent confirms chaos. Calculation of these exponents requires a signal of long duration. In addition, these exponents are very sensitive to noise influence. This makes them unsuitable for classification of signals with fast variations in time. Chaos detection based on topological and information measures of attractors reconstructed from available data is analyzed in [9]. Alternative techniques, based on detection of nonlinearity and short-term predictability, are proposed in [10]-[12]. The main disadvantage of these techniques is calculation complexity.

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Nonlinear oscillatory circuits are an important group of nonlinear systems that could have chaotic behavior. In periodic regime these circuits produce signals that can be represented as a sum of several sinusoidal components, i.e., as a sum of Dirac pulses in spectral domain. In a chaotic regime, numerous additional components in spectral domain can be observed and signal spectrum is broadband and noise-like. This difference between spectral content of signals from nonlinear chaotic oscillators is used in [1], where an efficient algorithm for detection of chaotic states in nonlinear oscillatory circuits has been recently proposed. The proposed detector is based on the specific measure of concentration of time-frequency (TF) signal representation. The short-time Fourier transform (STFT) has been used in [1] for design of the detector but other TF or time-scale distributions can be used with the same or slightly different algorithm setup. Here, results obtained using the S-method [13], [14], are demonstrated. An interesting favorable property of this detector is a fact that its application is not limited to nonlinear oscillators, since similar behavior of signals in spectral domain can be observed in some other common chaotic systems such as Lorenz, Rossler, Duffing and logistic map.

Influence of the window width, applied in the calculation of the TF representations, to the detector accuracy for noisy environment is analyzed in this paper. It has been shown that a wider window in the TF representations produces results robust to noise influence. However, a wide window can produce wrong classification of instants close to the periodic regime border. Narrower window produces an opposite behavior. The influence of noise is more emphatic for narrow windows, while samples close to the periodic state borders are classified with higher accuracy. This suggests existence of an optimal window width for a given circuit and noisy environment. Optimal window could accurately classify samples from the periodic regime close the border under influence of moderate amount of additive noise. However, determination of the optimal window width is not a trivial task. Instead of an elaborate procedure for determination of the optimal window width, we propose here a multiple STFTs-based approach for detection of chaotic states. In this algorithm the STFT-based detector of the chaotic state is applied with different window widths used in the STFT. Results obtained with different STFTs are combined in order to classify samples of the signals from chaotic oscillators. This is different from the traditional multiwindow techniques [15]-[18], where feature of signals are extracted from a TF representation obtained as a weighted sum of the STFT calculated with different window functions. The proposed approach outperforms the constant window size detectors in our experiments.

The paper is organized as follows. A brief overview of the STFT-based detector of chaotic state is presented in Section II. Influence of the applied window width is studied in Section III. Multiple STFTs-based approach is proposed in Section IV. Numerical examples and accuracy of detector study is presented in Section V. Concluding remarks are given in Section VI.

II. Detector based on the STFT - an overview

Here we briefly review the chaos detector based on the STFT proposed in [1]. By varying some of the circuit parameters chaotic oscillators can pass through different periodic and chaotic states. In periodic regime, chaotic oscillator produces signal that can be represented as a sum of several sinusoidal components. In chaotic regime, signal is broadband and noise like with numerous additional components. Details on spectral behavior of chaotic signals can be found in [19].

Detector proposed in [1] estimates oscillator state based on specific concentration measure of the part of the TF representation between the direct component and the main harmonic. The STFT, as a cross-term free TF representation, is used in [1] as a main tool in this detector. The STFT is defined as [20]:

$$STFT(t,f)=\int_{-\infty}^{\infty} x(t+\tau)w(\tau)e^{-j2\pi f\tau}d\tau, \quad (1)$$

where $x(t)$ is signal of interest (in this paper, it is a voltage or a current from the electric circuit), and $w(\tau)$ is the window function, where $w(\tau)=0$ for $|\tau|>T/2$ and T is window width. In order to avoid complex-valued STFT, we use its squared magnitude, i.e., the spectrogram $SPEC(t,f)=|STFT(t,f)|^2$. Within numerical study we will consider a bilinear representation from the Cohen class called the S-method that can be realized based on the STFT without signal oversampling [13], [14]:

$$SM(t,f)=$$
$$=\int_{-\infty}^{\infty} \Pi(\theta)STFT(t,f+\theta)STFT^{*}(t,f-\theta)d\theta. \quad (2)$$

A frequency window $\Pi(\theta)$ determines fundamental properties of the S-method. Namely, for $\Pi(\theta)=\pi\delta(\theta)$ the spectrogram follows. This is a cross-terms free (or reduced) representation, but signal components in the TF plane are not highly concentrated. However, a wide frequency window, $\Pi(\theta)=1$, produces the Wigner distribution with highly concentrated components but with emphatic cross-terms [13]. Then, window of relatively small width $\Pi(\theta)=1$ for $|\theta|\leq\Theta$, and $\Pi(\theta)=0$ can cause for $|\theta|>\Theta$, that we obtain highly concentrated signal components as in the Wigner distribution but without cross-terms.

We assume that a signal in periodic regime is represented with a sum of finite number of sinusoids (or signals with slight variations in frequency). However, the signal in the chaotic regime is broadband and noise like. It means that in spectral domain the signal content for periodic regime would be spread over the entire frequency domain. This consideration motivates the specific measure of chaotic state for signals from oscillatory circuits based on counting samples with high energy between

DC (direct current - frequency $f = 0$) and dominant frequency component (or just the first harmonic component). Note that the proposed algorithm is efficient and it can separate chaotic regime from the moderate amount of noise, as it will be shown in the simulations section.

The concentration measure of TF representations defined in [1] is given as:

$$m(t) = \int_{0}^{f_m(t)} u_{\Omega(t)}(t; f)df, \quad (3)$$

where $f_m(t)$ is a frequency of the main spectral component, determined as the position of the $TF(t, f)$ maxima:

$$f_m(t) = \arg\max_{f} TF(t, f). \quad (4)$$

Function $u_{\Omega(t)}(t; f)$ is given as:

$$u_{\Omega(t)}(t; f) = \begin{cases} 1 & TF(t, f) \geq \Omega(t) \\ 0 & \text{elsewhere}, \end{cases} \quad (5)$$

where threshold $\Omega(t)$ is selected in such a manner that values of the TF representation with magnitude greater than $\Omega(t)$ contain the given percentage of signal energy, typically $99.5 - 99.9\%$. Decision of the system state is made by comparing detector response function $m(t)$ with detector threshold $C(t)$:

$$d_T(t) = \\ = \begin{cases} 1 & \text{for} \quad m_T(t) \geq C_T(t) \quad \text{chaotic regime} \\ 0 & \text{for} \quad m_T(t) < C_T(t) \quad \text{periodic regime}, \end{cases} \quad (6)$$

where index T indicates that both detector response and threshold depend of the window width used in the STFT calculation. Note that both representations, the spectrogram and the S-method, are calculated using the STFT. Detection threshold $C_T(t)$ is calculated as an arithmetic mean between the expected detector responses in chaotic and periodic regimes. In chaotic regime we expect that the TF representation in the entire interval $[0, f_m(t)]$ is above the threshold $\Omega(t)$. Then, the expected detector response for this regime is $f_m(t)$. For the periodic regime we assume that only the main spectral and DC components are above $\Omega(t)$ in the considered interval, producing the expected detection response for this regime approximately $1.5F_w(T)$[1], where $F_w(T)$ is width of the main lobe of the window function in spectral domain that depends on the used window type $w(t)$ and its width T. Then, the detector threshold is:

$$C_T(t) = \frac{f_m(t) + 1.5F_w(T)}{2}. \quad (7)$$

The algorithm is summarized in Table I.

III. Influence of window width to detector accuracy

Influence of window width to the detector accuracy will be considered within a demonstrative example. Let us consider the time instant t_0 that is close to the periodic regime border in Fig.1. The wide window (T_2) contains samples from both periodic and chaotic regimes. This causes that the detector response function $m_T(t)$ increases and if it increases above the detector threshold $C_T(t)$ this sample can be misidentified as chaotic. However, narrow window (T_1) contains only samples from periodic regime. Then, for narrow window instant t_0 would be properly identified as being within periodic regime. From this analysis follows that the narrow windows are optimal for periodic regime detection. However, this conclusion is valid only for non-noisy environment. Note that a very important property of chaos detection systems is to distinguish chaos regime from noise. Namely, it has been shown in [21] that ability of feature extraction from the STFT for noisy environment improves with increasing the window width. This behavior is caused by increasing the ratio between the amplitude of signal components in the TF representation to the noise variance by increasing the width of the used window function [21].

Based on this brief analysis we can conclude that in noisy environment there is the window width producing a trade-off between robustness to the noise influence and proper identification of circuit state. However, determination of the optimal window width for circuit

[1]Half of DC is in negative frequency region.

TABLE I

Summary of the algorithm for chaotic behavior detection.

Step 1.	**Calculation of the TF representation.**
Step 2.	**Dominant spectral component frequency estimation.** Position of the TF representation maximum is adopted as a dominant frequency $f_m(t) = \arg\max_f TF(t,f)$.
Step 3.	**Determination of the threshold** $u_{\Omega(t)}$. **3a.** Sort the samples of the TF representation in order to obtain a sequence with decreasing magnitudes. **3b.** Threshold is selected as a value of the TF representaion such that the remaining (smaller) samples of the TF in the considered instant produce energy less than $\varepsilon \int_0^\infty TF(t,f)df$. In our experiments $\varepsilon = 0.0025$ is selected.
Step 4.	**Calculation of detector response function.** **4a.** Calculation of $m(t)$ using (3). **4b.** Averaging in the local neighborhood: $m'(t) = \frac{1}{p}\int_{t-p/2}^{t+p/2} m(\tau)d\tau$. We selected $p = 0.15$ms.
Step 5.	**Determination of current state by** (6)**.**

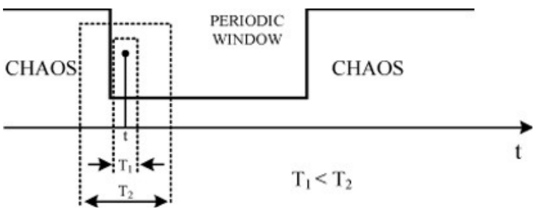

Fig. 1. Illustration of the STFT calculation for single time instant t_0 within periodic regime and two different window widths.

state estimation is very difficult. Instead of a direct search for the optimal window width, we propose a multiple STFTs-based (or multiple TF representations-based) approach in the next section, in order to achieve high detector accuracy for instants close to the limits of intervals with robustness to influence of moderate additive noise amount.

IV. Multiple STFTs-based detector

Both considered effects, noise and taking chaotic samples within window, increase the detector response function $m_T(t)$ in the periodic regime. They can cause that periodic samples be misidentified as chaotic. However, these effects have opposite behavior for very narrow and wide windows. We expect that for window widths close to optimal these effects will not disturb detector accuracy, and that circuit state will be correctly identified. Then, we can assume that a considered instant is in periodic regime if any window width produces detector response function indicating periodic state. For this aim we consider the set of window widths:

$$\mathbf{T} = \{T_i = a^i T_0 \mid i \in [0, Q]\}, \tag{8}$$

where $a > 1$ and T_0 is assumed to be very narrow window (producing accurate results for instants close to regime borders but with emphatic noise influence), while the widest window from the set T_Q has suppressed noise influence. Geometric progression of window widths is commonly used for estimation purpose in signal processing [22]. The narrowest window should be selected in such a manner that the DC and main spectral components are separated for more than the width of signal components in frequency domain, i.e., $f_m(t) > 1.5F_w(T)$. The widest window from the set is selected in such a manner that magnitude of signal components is at least an order of magnitude larger than variance of noise influence. For this purpose we use derivations from [21]. The STFT for each window from the set is calculated producing the corresponding detector response function $m_{T_i}(t)$, $i \in [0, Q]$. The next step is classification based on functions $d_{T_i}(t)$, eq.(6). Finally, decision if the current instant belongs to chaotic or periodic regime is made as:

$$d(t) = \\ = \begin{cases} 0 & \sum_{i=0}^{Q}(1-d_{T_i}(t)) \geq p \quad \text{periodic regime} \\ 1 & \text{elsewhere} \quad \text{chaotic regime}, \end{cases} \tag{9}$$

where p is an integer $1 \leq p < Q$. In order to avoid situation that a single window from the set indicates periodic regime $p > 1$ is used in our experiments. This is very important in order to keep accurate estimation of chaotic state, as it will be shown in the next section.

V. Numerical study

We consider the Chua's circuit given in Fig.2a with the function of the corresponding non-linear element given in Fig.2b, within so called period doubling route to chaos (for details related to this circuit and route to chaos see [19]). This route to chaos is produced by varying parameter G (conductance of a linear resistor from the circuit). Increase of G causes that circuit moves from periodic to chaotic regime. However, numerous periodic regions could exist within chaotic regime. In our case we consider the Chua circuit with the following set of parameters: $L_1 = 18$mH, $C_1 = 10$nF, $C_2 = 100$nF, $G_a = -757.576\mu$S, $G_b = -409.091\mu$S, $E = 1V$, $R_0 = 12.5\Omega$. Periodic and chaotic regimes are created by varying the parameter G. For $G = 530.12\mu$S we have the circuit in the periodic regime while for $G = 565.12\mu$S the chaotic regime follows. Within the signal duration one may observe numerous periodic regions varying lengths from 81 to 1552 samples (sampling rate $\triangle t = 12.6\,\mu$s). Here we consider the periodic region in the interval $t \in [12, 16]$ms. This region has approximately 311 samples. The spectrogram obtained with relatively narrow window of 180 samples is depicted in Fig.3a, with the region of interest zoomed in Fig.3b. The spectrogram with the wide window of 357 samples is depicted in Fig.3d, with zoomed part in Fig.3c. It can be seen that the proximity to chaotic regime increases the value of the TF representation within the periodic regime. This effect is more emphatic for the spectrogram with the wide window.

The set of window widths used in the algorithm is: $T_0 = N_0 \triangle t$ with $N_0 = 128$, $a = 1.121$, and $Q = 9$. Fig.4, left column, presents results obtained with 4 different window widths from the considered set for non-noisy case for considered periodic region. Note that the detector response in Fig.4b corresponds to the TF representation from Fig.3b, while the detector response for a wide window in Fig.4d corresponds to the TF representation from Fig.3c. Dashed lines in Fig.4 represent detection threshold $C_{T_i}(t)$. Note that an ideal detector would have classified the entire considered interval as periodic (detector response function below the threshold). However, here we can see that instants close to the window border are misidentified as chaotic state. The narrowest window produces the best results in this case with the smallest number of misidentified samples. This effect is the most emphatic for the widest window. Fig.4, right column, depicts detector response function for Gaussian noisy environment with $SNR = 10$dB. Definition of SNR from the Donoho's paper is applied here [1], [23]:

$$SNR = 10\log_{10}\frac{\int_t |f(t)-\overline{f}(t)|^2 dt}{\int_t |\nu(t)|^2 dt} \tag{10}$$

where $\overline{f}(t)$ is the signal mean value in the short interval around the considered instant:

$$\overline{f}(t) = \frac{1}{\eta}\int_{t-\eta/2}^{t+\eta/2} f(\tau)d\tau. \tag{11}$$

In our simulation parameter η is selected to be 3ms.

Here we can see that the narrowest window (that is the best for non-noisy case) produces the worst results. We compared accuracy of the proposed algorithm with $p = 1$, $p = 2$ and $p = 3$, with the algorithm with constant window widths in the STFT. Here, we also use the S-method in order to show that other TF representations can produce some benefits for this application. The S-method is calculated with (2) where the frequency window width has

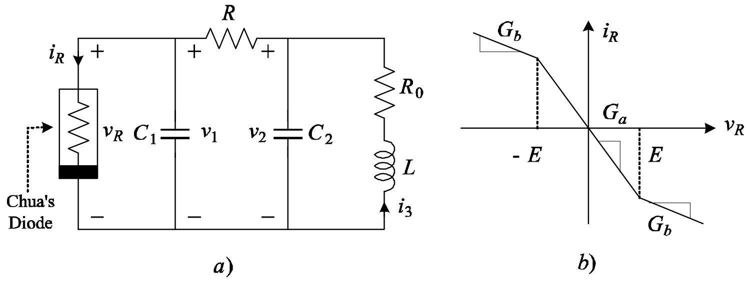

Fig. 2. (a) Chua's circuit; (b) nonlinear *v-i* characteristic of Chua's diode.

been set to $\Theta_i = 1/T_i$, i.e., one frequency sample for each considered window width. Note that increasing of the frequency window width causes appearance of the cross-term between DC and dominant frequency components. Results are depicted in Fig.5. It can be seen that for high SNR the narrowest window produces better results than other (constant width) windows. However, performance of these windows deteriorate rapidly with increasing the noise influence. The proposed algorithm outperforms the constant window widths for almost entire considered interval of SNR. The best results are produced for $p = 1$, and they are slightly better than with $p = 2$ and $p = 3$. In addition, the S-method with $p = 3$ produces very stable results with respect to the noise influence. These results are slightly worse than the corresponding results for the STFT-based estimator for high SNR but for small SNR the S-method significantly outperforms the STFT.

Finally, percentage of misidentified samples in chaotic regime for various p and SNR is considered with results given in Table II. It may be observed that the proposed detector produces better results for this regime for large p. Namely, if we set $p = 1$ just one mistake in detection of the chaotic regime causes that an instant be recognized as within periodic regime. Then we expect that increasing of p produces more accurate estimation of the chaotic regime that is confirmed by simulations. Note that a noise is "helpful" for identification of chaotic regime, since it increases the detector response function. It can be seen from Table II that for $p = 3$ and $SNR < 18$dB we have not made any mistake in identification of chaotic regime. Since $p = 3$ produces similar results as $p = 1$ and $p = 2$ for periodic regime, we recommend that $p = 3$, as a trade-off between accuracy in periodic and chaotic regimes, be used in this and similar experiments. Here it can be seen that the S-method produces higher error in detection of the chaotic regime than the corresponding STFT-based detector.

TABLE II
PERCENTAGE OF DETECTION ERRORS FOR SIGNAL CORRUPTED BY GAUSSIAN NOISE WITHIN CHAOTIC REGION.

SNR	p=1	p=2	p=3	SM p=3
20dB	5.9%	4.42%	1.89%	5.15%
18dB	4.7%	4.1%	1.26%	5.12%
16dB	4.1%	2.52%	0%	5%
14dB	3.15%	0.63%	0%	4.67%
12dB	1.89%	0%	0%	4.63%
10dB	0%	0%	0%	4.33%
8dB	0%	0%	0%	4.27%

VI. Conclusion

An approach for estimation of circuit state in nonlinear oscillators based on the multiples STFTs is proposed. The proposed technique retains the favorable properties of both narrow and wide windows used in the STFT. It produces better accuracy than any STFT-based detector, with constant window width. It has been demonstrated that the proposed detector can be applied to other (cross-term free or reduced) TF representations. The proposed

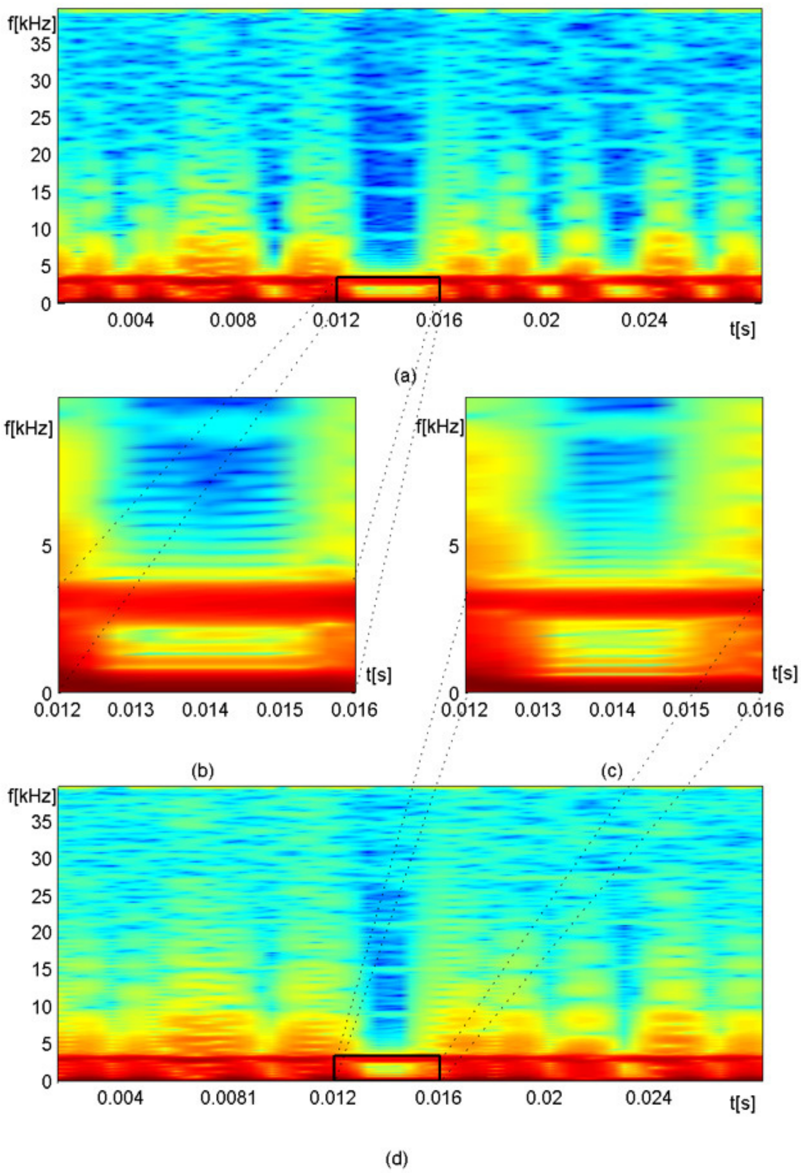

Fig. 3. STFT of the signal from chaotic circuit: (a) STFT obtained with relatively narrow window of 180 samples (zoomed region is given in (b)); (d) STFT obtained with relatively wide window of 357 samples (zoomed region is given in (c)).

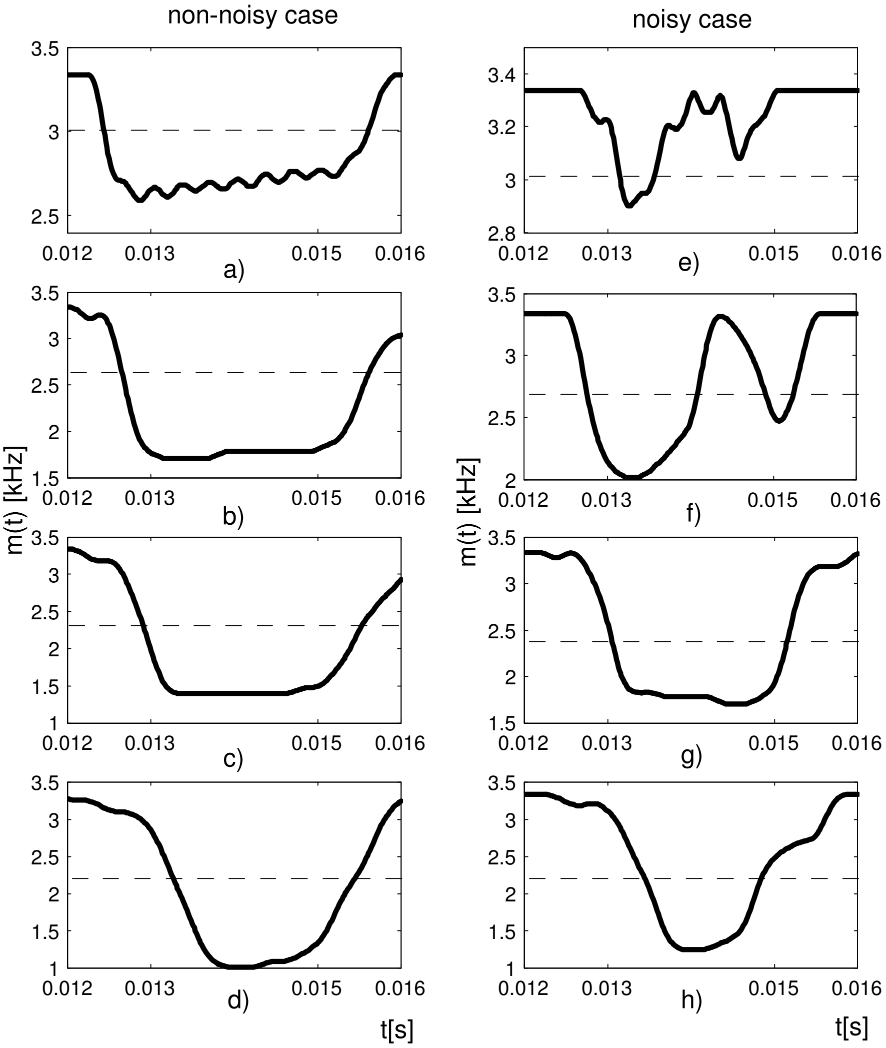

Fig. 4. Detector response in case of periodic region width $N = 311$ samples ($N \triangle t = 0.004$s): Left column-non-noisy environment; Right column-noisy environment ($SNR = 10$dB); a), e) $N_W = 128$; b), f) $N_W = 180$; c), g) $N_W = 254$; and d), h) $N_W = 357$.

detector based on the S-method produces significantly better results for high noise than the STFTs-based detector but with slightly worse results for small SNR and for detecting chaotic regime. This accuracy has been paid by increased calculation complexity. We will concentrate our future efforts to find a way to reduce number of required STFTs in the algorithm. In addition, optimization of the threshold level $C_T(t)$ will be considered in order to further decrease detection error in both chaotic and periodic states. More thorough study of other TF and time-scale distributions application in this research is also important step of further research.

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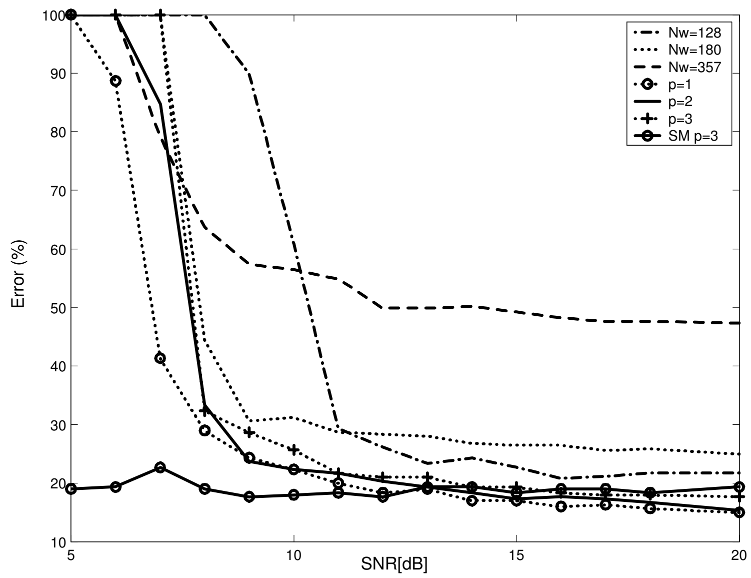

Fig. 5. Percentage of detector error with the proposed algorithm ($p = 1$, $p = 2$, $p = 3$) and with constant window widths for the STFT-based detector, and with the S-method (SM $p = 3$).

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