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Sensor array signal tracking using a data-driven window approach

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Abstract

In many practical source tracking applications, the interval of source stationarity may severely vary with time, so that array observations may contain both almost stationary data blocks and nonstationary data intervals with rapidly moving sources. Moreover, typical situations may occur where some sources move rapidly within the window exploited, whereas the motion of the other sources is weak. In such scenarios, the traditional fixed-window approach appears to be nonoptimal because it may lead to a very poor tracking performance. Below, we address the narrowband direction of arrival (DOA) tracking problem using a new adaptive-window approach. In our technique, a separate data-driven window is used for each source of interest. The optimization of window lengths is based on the bias-to-variance tradeoff. The comparison of our approach with conventional fixed-window algorithms is presented showing that the underlying idea has an evident potential in nonstationary scenarios with rapidly moving sources. A natural price for the improved tracking performance is a higher computational cost and the restriction of our approach by the scenarios with 'well-separated' sources. © 2000 Elsevier Science B.V. All rights reserved.

Zusammenfassung

In vielen praktischen Anwendungen der Quellen-Nachführung können die Stationaritätsintervalle der Quellen zeitlich stark schwanken. Array-Beobachtungen können daher sowohl beinahe stationäre Datenblöcke als auch instationäre Datenblöcke mit schnell bewegten Quellen enthalten. Darüber hinaus können typische Situationen auftreten, bei denen sich einige Quellen innerhalb des Beobachtungsfensters schnell bewegen, während die Bewegung der anderen Quellen nur schwach ist. In solchen Szenarien scheint der traditionelle Ansatz eines festen Beobachtungsfensters nicht optimal zu sein, da er zu sehr schlechten Nachführungsergebnissen führen kann. In dieser Arbeit behandeln wir das Problem der Nachführung von Einfallsrichtungen (DOAs) im Schmalband-Fall, wobei wir einen neuen Ansatz mit adaptiven Fenstern verwenden. Bei unserer Methode wird für jede relevante Quelle ein eigenes datengesteuertes Fenster verwendet. Die Optimierung der Fensterlängen beruht auf dem Bias-Varianz-Austausch. Ein Vergleich unseres Ansatzes mit konventionellen Algorithmen mit festen Fenstern zeigt, dass die ihm zugrundeliegende Idee ein offensichtliches Potential für instationäre Szenarien mit schnell bewegten Quellen besitzt. Der Preis für die verbesserten

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Nachführungseigenschaften sind ein höherer Rechenaufwand und die Beschränkung unseres Ansatzes auf Szenarien mit "gut getrennten" Quellen.

Résumé

Dans de nombreuses applications pratiques de suivi de sources, l'intervalle durant lequel la source est stationnaire peut varier sévèrement au cours du temps, de sorte que des observations en réseau peuvent contenir à la fois des blocs de données presque stationnaires et des intervalles de données non stationnaires, avec des sources bougeant rapidement. De plus, des situations typiques peuvent arriver où certaines sources bougent rapidement durant la fenêtre exploitée alors que le mouvement d'autres sources est faible. Dans de tels scénarios, l'approche traditionnelle par fenêtre fixe apparaît non optimale parce qu'elle peut mener à de très pauvres performances en suivi. Ci-dessous, nous adressons le problème de suivi par direction d'arrivée à bande étroite, en utilisant une nouvelle approche par fenêtre adaptative. Dans notre technique, une fenêtre séparée, dirigée par les données, est utilisée pour chaque source d'intérêt. L'optimisation de la longueur de la fenêtre repose sur un compromis biais/variance. Nous présentons la comparaison de notre approche avec les algorithmes conventionnels à fenêtre fixe, qui montre que l'idée sous-jacente a un potentiel évident pour des scénarios non stationnaires avec des sources bougeant rapidement. Le prix naturel pour des performances de suivi est un coût de calcul plus important et la restriction de notre approche par les scénarios avec des sources ⟪bien séparées⟫.

Keywords: Source tracking; Data-driven windows; Root-MUSIC

1. Introduction

In typical nonstationary array processing scenarios, the interval of data stationarity tends to vary with time, i.e., the received data may include both highly nonstationary and almost stationary blocks. Another typical situation occurs where some sources move rapidly within the window exploited, whereas the motion of the remaining part of sources is weak.

In such scenarios, the lag window length becomes one of the most important parameters. In the traditional fixed-window approach, the use of short windows is well known to increase the variance of direction finding techniques. With longer lag windows, the estimation variance can be lowered but the DOA estimates become biased and, therefore, are unable to track rapidly moving sources. As a result, the traditional fixed-window approach does not enable tracking multiple sources with severely different intervals of stationarity.

In this paper, we develop a new adaptive-window approach to DOA tracking. In our technique, multiple data-driven windows are used, i.e., a separate adaptive window is employed for each source. Our algorithm combines the developed adaptive multi-window subspace tracker and the popular root-MUSIC technique [1,12]. The adaptive-window selection procedure is based on the approximate minimization of the mean squared estimation error using the bias-to-variance tradeoff approach developed originally for another class of problems [7–9]. Comparisons with conventional fixed-window algorithms demonstrate a potential of the developed adaptive-window approach. A natural price for the improvements achieved is a higher computational cost. Also, our approach is restricted by scenarios with 'well separated' sources.

2. Signal model

Assume that a uniform linear array (ULA) of n sensors receives q $(q < n)$ narrowband signals impinging from the unknown varying directions $\{\theta_1(t), \theta_2(t), \ldots, \theta_q(t)\}$. The output vector of the array at the discrete time t can be expressed as

$$\mathbf{x}(t) = \mathbf{A}(t)\mathbf{s}(t) + \mathbf{n}(t), \tag{1}$$

where the $n \times q$ time-varying direction matrix

$$\mathbf{A}(t) = [\mathbf{a}_1(t), \mathbf{a}_2(t), \ldots, \mathbf{a}_q(t)] \tag{2}$$

is composed of the source direction vectors

$$\mathbf{a}_i(t) = \left(1, \exp\left\{\mathrm{j}\frac{2\pi}{\lambda} d \sin\theta_i(t)\right\}, \dots, \exp\left\{\mathrm{j}\frac{2\pi}{\lambda} d(n-1)\sin\theta_i(t)\right\}\right)^{\mathrm{T}}, \tag{3}$$

λ is the wavelength, d is the interelement spacing, $(\cdot)^{\mathrm{T}}$ stands for the transpose, and the $q \times 1$ and $n \times 1$ vectors $\mathbf{s}(t)$ and $\mathbf{n}(t)$ contain the source waveforms and the sensor noise, respectively.

3. Conventional fixed-window approach

In this section, we revisit the traditional fixed-window approach with the rectangular sliding window containing M independent data snapshots.[2] Write the data matrix as

$$\mathbf{X}(t) = [\mathbf{x}(t - M/2), \mathbf{x}(t - M/2 + 1), \dots, \mathbf{x}(t + M/2 - 1)]. \tag{4}$$

The lag window estimate of the array covariance matrix

$$\mathbf{R}(t) = E\{\mathbf{x}(t)\mathbf{x}^{\mathrm{H}}(t)\} = \mathbf{A}(t)\mathbf{S}(t)\mathbf{A}^{\mathrm{H}}(t) + \sigma^2 \mathbf{I} \tag{5}$$

is given by

$$\hat{\mathbf{R}}(t) = \frac{1}{M}\mathbf{X}(t)\mathbf{X}^{\mathrm{H}}(t), \tag{6}$$

where $\mathbf{S}(t) = E\{\mathbf{s}(t)\mathbf{s}^{\mathrm{H}}(t)\}$, $\mathbf{I}$ is the identity matrix, σ^2 is the sensor noise variance, and $(\cdot)^{\mathrm{H}}$ stands for the Hermitian transpose.

Write the eigendecomposition of (6) as

$$\begin{aligned}\hat{\mathbf{R}}(t) &= \hat{\mathbf{E}}(t)\hat{\mathbf{\Lambda}}(t)\hat{\mathbf{E}}^{\mathrm{H}}(t)\\ &= \hat{\mathbf{E}}_{\mathrm{S}}(t)\hat{\mathbf{\Lambda}}_{\mathrm{S}}(t)\hat{\mathbf{E}}_{\mathrm{S}}^{\mathrm{H}}(t) + \hat{\mathbf{E}}_{\mathrm{N}}(t)\hat{\mathbf{\Lambda}}_{\mathrm{N}}(t)\hat{\mathbf{E}}_{\mathrm{N}}^{\mathrm{H}}(t),\end{aligned} \tag{7}$$

where the $q \times q$ and $(n-q)\times(n-q)$ diagonal matrices $\hat{\mathbf{\Lambda}}_{\mathrm{S}}$ and $\hat{\mathbf{\Lambda}}_{\mathrm{N}}$ contain the q and $n-q$ sample signal- and noise-subspace eigenvalues, respectively, whereas the columns of the $n \times q$ and $n \times (n-q)$ matrices $\hat{\mathbf{E}}_{\mathrm{S}}$ and $\hat{\mathbf{E}}_{\mathrm{N}}$ contain the sample signal- and noise-subspace eigenvectors, respectively.

[2] Without loss of generality, M is assumed to be even.

Note that there are many computationally efficient algorithms for updating the matrix $\hat{\mathbf{E}}_{\mathrm{S}}(t)$ (for example, see [2,17] and references therein). The discussion on what algorithm is better is beyond the scope of our study. Hereafter, assume that one of existing subspace tracking techniques is exploited. The last step of the fixed-window DOA tracker is to estimate the source DOA's, for example, using the root-MUSIC polynomial [1]

$$\begin{aligned}f_{\mathrm{MUSIC}}(z) &= \mathbf{a}^{\mathrm{T}}(1/z)\hat{\mathbf{E}}_N(t)\hat{\mathbf{E}}_N^{\mathrm{H}}(t)\mathbf{a}(z),\\ &= \mathbf{a}^{\mathrm{T}}(1/z)\{\mathbf{I} - \hat{\mathbf{E}}_{\mathrm{S}}(t)\hat{\mathbf{E}}_{\mathrm{S}}^{\mathrm{H}}(t)\}\mathbf{a}(z),\end{aligned} \tag{8}$$

where, according to (3)

$$\mathbf{a}(z) = [1, z, \dots, z^{n-1}]^{\mathrm{T}}, \quad z(t) = \exp\left\{\mathrm{j}\frac{2\pi}{\lambda} d \sin\theta(t)\right\}. \tag{9}$$

The estimates of source trajectories $\hat{\theta}_i(t)$, $i = 1, 2, \dots, q$ can be found from the roots of (8) in a standard way [1,12].

In the presence of the coherent (multipath) sources, the spatial smoothing algorithm can be incorporated in the tracker scheme [14].

4. Adaptive-window approach

Let us make the following assumptions:

(A1) The array is large ($n \gg 1$) so that the sources are well separated in the sense of Rayleigh criterion [3].

(A2) The source powers are subject to much slower variations than their DOA's.

(A3) The number of sources is known.

The first assumption is almost always true for large arrays. Although the high-resolution root-MUSIC algorithm will be exploited for DOA tracking, we stress that this algorithm is chosen because of other reasons than its high-resolution property. The motivation of this choice is due to a very simple implementation of root-MUSIC which is based on the eigendecomposition of the array covariance matrix and polynomial rooting. It is worth noting that in the case $q \ll n$, the eigendecomposition can be performed using computationally efficient fast algorithms [16], and the computational cost of polynomial rooting is negligible as compared to that of the eigendecomposition if the fast Jenkins–Traub or Lang–Frenzel algorithms are employed [10].

Assumptions (A2) and (A3) are also very typical for array processing [15]. Denoting $\omega_i(t) = \sin\theta_i(t)$ and $\hat{\omega}_i(t) = \sin\hat{\theta}_i(t)$, and assuming that the sources are sorted so that $\omega_1 < \omega_2 < \cdots < \omega_q$ and $\hat{\omega}_1 < \hat{\omega}_2 < \cdots < \hat{\omega}_q$, let us obtain the optimal window length by minimizing the mean squared error (MSE) given by

$$\varepsilon_i^2(M) = \mathrm{bias}_i^2(M) + \mathrm{var}_i(M), \tag{10}$$

where

$$\mathrm{bias}_i = E\{\hat{\omega}_i - \omega_i\}, \quad \mathrm{var}_i = E\{(\hat{\omega}_i - E\{\hat{\omega}_i\})^2\}, \tag{11}$$

and the explicit dependence on the window length is emphasized. Here, we stress that in what follows, we formulate our algorithm in terms of spatial frequencies ω_i rather than the DOA's θ_i.

Note that the bias of any DOA estimate cannot be known a priori because it depends on unknown source motion parameters (i.e., on the angles $\theta_i(t)$ and their derivatives). Furthermore, in the nonstationary case, the bias component is mainly determined by the rapid DOA changes within the sliding window rather than the finite sample effects. In [7], an elegant approximate solution minimizing (10) has been presented, based solely on the variance knowledge. This approach is usually referred to as an intersection of confidence intervals (ICI) criterion [7,8]. Below, this solution is adapted to the problem considered. The key idea of the ICI criterion is to find the optimal window length from the bias-to-variance tradeoff, i.e., from the condition that the bias squared should have the same order of magnitude as the variance.

It can be shown [15] that under Assumption (A1) and in the stationary case, the variance of the spatial frequency $\hat{\omega}_i$ at the output of the spectral MUSIC estimator can be expressed as

$$\mathrm{var}_i \simeq \frac{6\lambda^2(1 + 1/n\,\mathrm{SNR}_i)}{M(2\pi d)^2\,\mathrm{SNR}_i\, n(n^2-1)}, \tag{12}$$

where $\mathrm{SNR}_i = \sigma_i^2/\sigma^2$, and $\sigma_i^2 = \mathbf{S}_{i,i}$ is the variance of the ith source waveform. Rao and Hari [12] have shown that the same expression (12) is valid for the root-MUSIC technique as well. It is worth noting that (12) does not depend on the source spatial frequency ω_i, i.e., it depends only on the unknown source SNR and known wavelength λ, interelement spacing d, number of sensors n, as well as the chosen window length M. It is also important to note that (12) is a large sample ($\mathrm{O}(1/M)$) approximation. However, in situations with well-separated sources, even a few snapshots are already sufficient to approach the value of (12). To illustrate this property, we display in Fig. 1 the ratio C of the experimental and theoretical root-mean-square errors (RMSE's) versus M for a ten-element ULA, two equipowered sources with the DOA's $\theta_1 = 0^\circ$ and $\theta_2 = 20^\circ$, and SNR = 10 dB. The experimental RMSE was computed using 1000 independent runs and averaged over the sources. From Fig. 1, we observe that the parameter C rapidly converges to $C = 1$, and expression (12) becomes valid with a good precision starting from M = 4–8 snapshots.

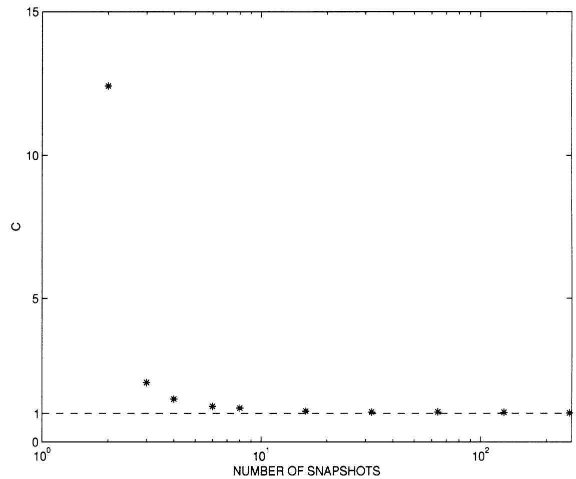

Fig. 1. The ratio C of the experimental and theoretical RMSE's versus M.

Let us restrict the absolute value of the estimation error by

$$|\omega_i - \hat{\omega}_i(M)| \leqslant |\mathrm{bias}_i(M)| + \kappa\sqrt{\mathrm{var}_i(M)}, \tag{13}$$

where the distribution of the estimate $\hat{\omega}_i$ is assumed to be Gaussian [15], and (13) holds with the probability $P(\kappa)$ for the corresponding quantile κ of the standard Gaussian distribution $\mathcal{N}(0,1)$. Let the window length M be so small that [7,9]

$$|\mathrm{bias}_i(M)| \leqslant \kappa\sqrt{\mathrm{var}_i(M)}, \tag{14}$$

Using (14), Eq. (13) can be rewritten as

$$|\omega_i - \hat{\omega}_i(M)| \leqslant 2\kappa\sqrt{\mathrm{var}_i(M)}. \tag{15}$$

Let us now consider a discrete set of window lengths $\{M_l\}_{l=1}^{K}$. If these window lengths provide such a small bias then the segments

$$D_l = [\hat{\omega}_i(M_l) - 2\kappa\sqrt{\mathrm{var}_i(M_l)},\ \hat{\omega}_i(M_l) + 2\kappa\sqrt{\mathrm{var}_i(M_l)}],\ l = 1,2,\ldots,K \tag{16}$$

have a common point (i.e., intersect with each other). Condition (15) becomes violated if some window lengths from the set $\{M_l\}_{l=1}^{K}$ produce strongly biased estimates, so that $|\mathrm{bias}_i(M)| > \kappa\sqrt{\mathrm{var}_i(M)}$. Therefore, to find a reasonable approximation to the optimal window length, it is meaningful to exploit (16) referred to as the ICI criterion [7]. Obviously, this criterion corresponds to the following bias-to-variance tradeoff condition:

$$|\mathrm{bias}_i(M)| \simeq \kappa\sqrt{\mathrm{var}_i(M)}. \tag{17}$$

The discrete set $\{M_l\}_{l=1}^{K}$ can be thought to be a grid covering the window lengths of interest.

According to (17), the essence of the exploited tradeoff is to compare the empirical bias with the variance predicted using (12). Since the parameter κ should be chosen so that $\kappa \sim 1$, the underlying tradeoff compares *the orders of magnitude* of the bias and variance rather than the exact values. This is the reason why the ICI criterion provides a sufficient degree of robustness against possible variance estimation errors [8,9].

The estimates of the confidence intervals (16) can be written as

$$\hat{D}_l(t) = [\hat{\omega}_i(M_l,t) - 2\kappa\sqrt{\widehat{\mathrm{var}}_i(M_l,t)}, \hat{\omega}_i(M_l,t) + 2\kappa\sqrt{\widehat{\mathrm{var}}_i(M_l,t)}], \quad l = 1,2,\ldots,K, \tag{18}$$

where

$$\widehat{\mathrm{var}}_i(M_l,t) = \frac{6\lambda^2(1 + 1/n\widehat{\mathrm{SNR}}_i(t))}{M_l(2\pi d)^2\widehat{\mathrm{SNR}}_i(t)n(n^2-1)} \tag{19}$$

is the estimate of (12), and $\widehat{\mathrm{SNR}}(t)$ is any estimate of the SNR of the ith source. In what follows, the source SNR's will be estimated as

$$\widehat{\mathrm{SNR}}_i(t) = \frac{1}{(2P+1)\hat{\sigma}^2}\sum_{p=t-P}^{t+P} \max{}_i(h(p,\theta)), \tag{20}$$

where

$$h(p,\theta) = \frac{|\mathbf{a}^{\mathrm{H}}(\theta)\mathbf{x}(p)|^2}{n^2}, \tag{21}$$

$\hat{\sigma}^2$ is an estimate of the noise variance, $\max_i(\cdot)$, $i = 1,2,\ldots,q$ are the q highest maxima of (21) sorted with respect to the source index i, and $2P+1$ is the length of the estimating interval. Estimate (20) corresponds to the averaged outputs of the single-snapshot conventional beamformer which can be exploited here according to Assumptions (A1) and (A2). The estimate $\hat{\sigma}^2$ of the noise variance can be found using several reliable ways, for example, by averaging the minimal eigenvalues of the covariance matrix, using single-snapshot deconvolution procedures (such as a popular CLEAN algorithm [13]), or by means of calibration measurements carried out in advance (in the absence of signal sources). According to Assumption (A2), $P \gg \min\{\{M_l\}_{l=1}^{K}\}$ can be chosen to stabilize the estimates (20) and $\hat{\sigma}^2$. It is worth noting that even rather approximate estimates (i.e., up to the order of magnitude) of the source SNR's suffice for the ICI criterion [7,9]. It is also important that the particular estimate (21) is a slight modification of a single-snapshot variant of the maximum likelihood (ML) estimate of the signal power derived in [6,4]. Therefore, it can be expected to have a sufficiently high performance. Note also that the estimate (20) is biased in the general case but its bias becomes negligible in large arrays (Assumption (A1)) [6].

Now, we formulate our adaptive-window DOA tracking algorithm as the following sequence of steps:

Step 1: Specify a sequence of window lengths (sorted in ascending order)

$$\mathscr{M} = \{M_l\}_{l=1}^{K}, \quad M_1 < M_2 < \cdots < M_K. \tag{22}$$

For each value M_l, compute (or update) the output of the DOA tracker based on the root-MUSIC polynomial (8). As a result of this step, we get the sorted estimates $\hat{\omega}_1(M_l) < \hat{\omega}_2(M_l) < \cdots < \hat{\omega}_q(M_l)$

of spatial frequencies $\hat{\omega}_i(M_l)$ obtained for each source $i = 1, \ldots, q$ and each window length $l = 1, \ldots, K$.

Step 2: For each source, estimate its SNR using (20), and then insert (20) into (19) to obtain the estimates $\widehat{\mathrm{var}}_i(M_l, t)$ for each source and each window length M_l. Using these estimated variances and (18), for each source find the estimates $\hat{D}_1(t), \ldots, \hat{D}_K(t)$.

Step 3: For each source, obtain the optimal window length $M_{\mathrm{opt}}(i,t)$ determined as the largest M_l from set (22) for which the estimated segments $\hat{D}_{l-1}(t)$ and $\hat{D}_l(t)$ still intersect (have a common point). In other words, we obtain the optimal window length via the largest index $l \in \{1, \ldots, K\}$ for which the following inequality:

$$|\hat{\omega}_i(M_l, t) - \hat{\omega}_i(M_{l-1}, t)| \leqslant 2\kappa\left(\sqrt{\widehat{\mathrm{var}}_i(M_l, t)} + \sqrt{\widehat{\mathrm{var}}_i(M_{l-1}, t)}\right) \tag{23}$$

is satisfied. If all intervals exploited do not intersect, the shortest window should be taken.

This step results in q optimal windows (one window per source) $M_{\mathrm{opt}}(i,t)$, $i = 1, \ldots, q$.

Step 4: Exploit the obtained optimal windows in the DOA tracker described in Section 3.

Note that in Step 3, a particular variant of the ICI criterion is used, based on the intersection of two neighboring confidence intervals [9].

It should be noted that after a proper modification, our approach can be applied to the exponential window case as well. However, to treat this case, another expression for the variance is required instead of (12).

5. Simulations

We have assumed a ULA of five omnidirectional sensors with the half-wavelength spacing. SNR = 1.25 dB has been assumed for each source in a single sensor. The simplest two-window algorithm was implemented with the window lengths equal to 8 and 128 snapshots (i.e., $M = 8, 128$). In all figures given below, the true source trajectories are indicated by dashed lines.

In the first two examples, we simulated the single source scenario. Fig. 2(a)–(c) displays the estimated

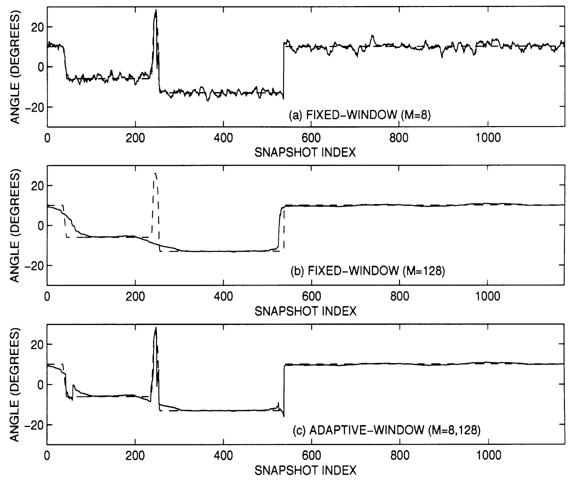

Fig. 2. Tracking performances of (a) the fixed-window algorithm with $M = 8$, (b) the fixed-window algorithm with $M = 128$, and (c) the adaptive-window algorithm in the first example. $\kappa = 2$. The true source trajectory is shown with a dashed line.

trajectories for the first example using the fixed-window algorithm with $M = 8$, the fixed-window algorithm with $M = 128$, and the adaptive-window algorithm, respectively. The parameter $\kappa = 2$ has been taken. Similar plots for the second example are displayed in Fig. 3(a)–(c). From this figure, we see that for $\kappa = 2$, the residual effect of bias is still quite essential in the interval between 120th and 350th snapshots. Decreasing κ, we are able to reduce this effect, but at the expense of higher variance. This is demonstrated in Fig. 4(a)–(c) which corresponds to the second example with the value $\kappa = 1.4$.

Additionally, the Empirical RMSE's (ERMSE's) [5,11]

$$\mathrm{ERMSE} = \sqrt{\frac{1}{qT}\sum_{t=1}^{T}\sum_{i=1}^{q}(\hat{\theta}_i(t) - \theta_i(t))^2} \tag{24}$$

of these techniques have been compared. In (24), T is the length of estimated source trajectory. The ERMSE characterizes instantaneous DOA estimation errors averaged over the interval T.

In subplots (a)–(c) of Fig. 2, the ERMSE is 2.18°, 4.10°, and 1.43°, respectively. In the similar subplots of Fig. 3, the ERMSE is 1.65°, 4.01°, and 1.48°, respectively. In the subplots of Fig. 4, this

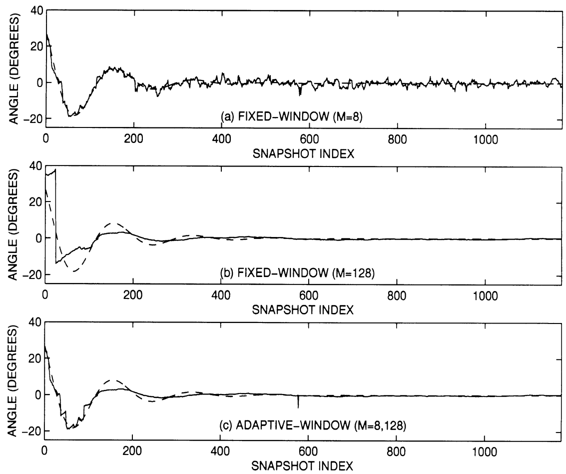

Fig. 3. Tracking performances of (a) the fixed-window algorithm with $M = 8$, (b) the fixed-window algorithm with $M = 128$, and (c) the adaptive-window algorithm in the second example. $\kappa = 2$. The true source trajectory is shown with a dashed line.

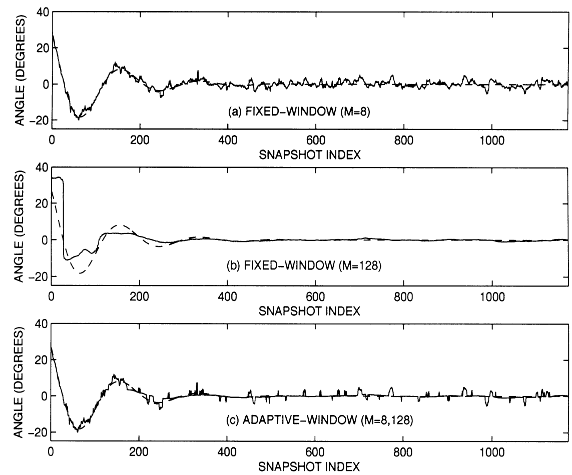

Fig. 4. Tracking performances of (a) the fixed-window algorithm with $M = 8$, (b) the fixed-window algorithm with $M = 128$, and (c) the adaptive-window algorithm in the second example. $\kappa = 1.4$. The true source trajectory is shown with a dashed line.

parameter is 1.65°, 4.01°, and 1.44°, respectively. We see that the adaptive-window algorithm has the smallest ERMSE among the techniques tested and finds an excellent tradeoff between the estimation bias and variance.

In the third example, the scenario with two uncorrelated sources was simulated and $\kappa = 2$ is taken. Fig. 5(a)–(c) shows the estimated trajectories for this example using the same techniques as in Figs. 2–4. In subplots (a)–(c) of this figure, the ERMSE is 2.32°, 4.13°, and 1.82°, respectively. As in the first and second examples, the adaptive-window algorithm performs better than both fixed-window techniques, i.e., has the smallest ERMSE.

Hence, from our simulations it follows that the presented adaptive-window algorithm has an obvious potential when applied to the source tracking problems in the presence of rapid and abrupt source trajectory changes. In particular, the proposed technique has reduced DOA estimation ERMSE's relative to the conventional (fixed-window) root-MUSIC based tracking algorithm. In fact, our algorithm provides more flexibility than the fixed-window approach because the use of multiple adaptive windows enables to track both slow

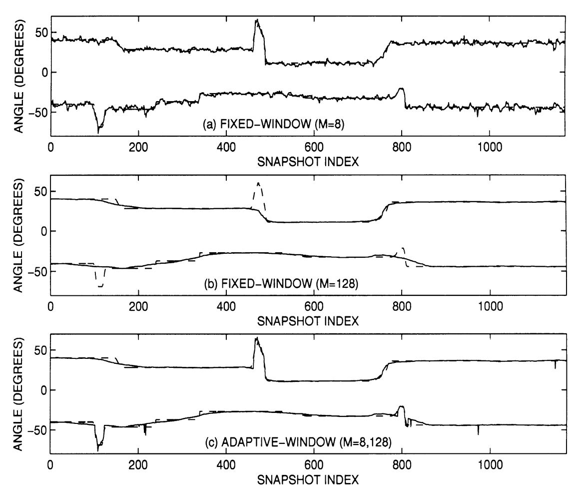

Fig. 5. Tracking performances of (a) the fixed-window algorithm with $M = 8$, (b) the fixed-window algorithm with $M = 128$, and (c) the adaptive-window algorithm in the third example. $\kappa = 2$. The true source trajectories are shown with dashed lines.

and fast (e.g. abrupt) trajectory changes. If in the multiple source case the motion of some sources is fast and that of the remaining sources is slow, the fixed-window algorithm may experience severe degradation. However, the multiwindow algorithm can easily treat this situation just by making use of multiple windows with different lengths (one window per source). This effect can be seen from Fig. 5 by examining the source tracking performance in the interval between 460th and 500th snapshots.

We end up this section with some remarks prompted by several additional simulations with more than two windows whose results were not detailed in this paper, in the interest of brevity.

The application of the algorithm with more than two windows to the examples considered showed further slight performance improvements which correspond only to several percents of reduction of the ERMSE. However, the multiwindow algorithm may have more significant improvements over the simplest two-window algorithm in more complex scenarios where the source trajectory has multiple scales of stationarity or where there is no a priori information for a motivated fixed-window choice. This issue requires more study in future.

6. Conclusions

In several practically important source tracking applications, the interval of source stationarity may vary with time, so that the array observations may contain both almost stationary and nonstationary data intervals. Even more complicated situations may occur where some sources move rapidly within the window, whereas the motion of the other sources is weak. In such scenarios, the traditional fixed-window approach may be nonoptimal because it may result in a significant degradation of the source tracking performance. The DOA tracking problem in the presence of high DOA nonstationarity and rapid (abrupt) source trajectory changes was addressed using the adaptive multiwindow framework. The so-called ICI approach (earlier developed for another class of problems) was adapted to the problem considered. The optimization of window length is based on the bias to variance tradeoff. A new DOA tracking algorithm with a data-driven (adaptive) window length was proposed. Comparisons with the conventional fixed-window source tracking algorithm demonstrated promises and feasibility of the new approach. A natural price for tracking improvements achieved is a higher computational cost and a restriction by "well separated" (low-resolution) source scenarios.

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