Robust S-transform based on L-DFT

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Abstract— **Time-frequency representations of signals obtained by the S-transform can be very sensitive to the presence of $\alpha-$stable noise. In this letter, an algorithm for the robust S-transform is introduced. The proposed scheme is based on the L-DFT. The results of conducted numerical analysis show a significantly enhanced performance of the proposed scheme in comparison to the standard S-transform.**

I. Introduction

Time-frequency representations (TFR) indicate variations of the signal spectral characteristics with time, and they are the natural choice as an analysis tool for the nonstationary signals [1]. The main objectives of the various types of time-frequency distribution functions (e.g., Wigner-Ville distribution (WVD) [1], short-time Fourier transform (STFT) [1], the S-transform [2]) are to obtain time-varying spectral density function with higher resolution, and to overcome any existing interferences [1].

In this paper, we address the performance of the S-transform for signals contaminated with $\alpha-$stable noise (e.g., [3]). Previous contributions only addressed the performance of traditional TFRs (e.g., the WVD and STFT in [4]), while the S-transform is a hybrid of short-time Fourier analysis and wavelet analysis. It employs variable window length producing higher frequency resolution at lower frequencies and sharper time localization for higher frequencies. This property of the S-transform makes it particularly suitable for the analysis of signals with hyperbolic or sinusoidally modulated components. In addition, the phase information provided by the S-transform is referenced to the time origin, and therefore provides supplementary information about spectra which is not available from locally referenced phase information obtained by the continuous wavelet transform [2]. For this reason, the S-transform has already been applied in many fields (e.g., [5]). Nevertheless, the S-transform exhibits very poor performance for signals contaminated with $\alpha-$stable noise (i.e., impulse noise). In order to resolve this issue we propose an algorithm for the robust S-transform based on the L-DFT. The results of the numerical analysis show that the proposed scheme achieves significantly enhanced performance.

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II. Proposed Scheme

The standard S-transform of a function $x(t)$ is given by a convolution integral as [2]:

$$S_x(t,f) = \int_{-\infty}^{+\infty} x(\tau)w(t-\tau,f)\exp(-j2\pi f\tau)d\tau \quad (1)$$

where a window function used in S-transform is actually a scalable Gaussian function defined as

$$w(t,f) = \frac{1}{\sigma(f)\sqrt{2\pi}}\exp\left(-\frac{t^2}{2\sigma^2(f)}\right) \quad (2)$$

with a constraint $\int_{-\infty}^{+\infty} w(t-\tau,f)d\tau = 1$. The advantage of the S-transform over the STFT is that the variance $\sigma(f)$ is actually a function of frequency, f, defined as $\sigma(f) = 1/|f|$. If only discrete samples of the continuous signal are available, the S-transform is implemented using the discrete Fourier transform (DFT) [2]. However, given that the standard DFT is sensitive to the impulse noise, then the representation obtained by the S-transform is greatly influenced by such noise. In order to alleviate these effects, we propose the implementation of the S-transform using the L-DFT [4]. The L-DFT is introduced as a trade-off between robustness to the impulse noise influence and quality of the spectra estimate.

Assume that we have N samples of $x(t)$ obtained with a sampling period T_s. Then, the robust estimate of the S-transform is given by

$$S_x(n,k) = \frac{1}{N}\sum_{m=0}^{N-1} X_L(k+m)$$

$$\times W\left(m,k\right)\exp\left(-j2\pi mn/N\right) \quad (3)$$

where $W\left(m,k\right)$ is the DFT of the discretized window function and $X_L\left(k\right)$ is given by

$$X_L\left(k\right) = \sum_{q=1}^{N-1} \beta_q \mathbf{r_{(q)}}\left(\mathbf{k}\right) + \mathbf{j}\sum_{\mathbf{q=1}}^{\mathbf{N-1}} \boldsymbol{\beta_q} \mathbf{i_{(q)}}\left(\mathbf{k}\right) \quad (4)$$

with $n, k(m) = 0,..,N-1$ representing discretized time and frequency; $\mathbf{r_{(q)}}\left(\mathbf{k}\right)$ and $\mathbf{i_{(q)}}\left(\mathbf{k}\right)$ belonging to the sets $\mathbf{R} = \{\mathrm{Re}\{\mathbf{Nx(n)}\exp\left(-\mathbf{j2\pi kn/N}\right)\} : \mathbf{n} \in [\mathbf{0}, \mathbf{N}-\mathbf{1}]\}$ and $\mathbf{I} = \{\mathrm{Im}\left\{\mathbf{Nx(n)}\exp\left(-\mathbf{j2\pi kn/N}\right)\right\} : n \in [0, N-1]\}$, respectively, sorted into the nonincreasing sequences. In this paper, we implement the trimmed mean form of the L-DFT [4], where β_q coefficients in the above equation are given as

$$\beta_q = \begin{cases} \frac{1}{4\gamma + N(1-2\gamma)} & q \in [(N-2)\gamma, \\ & \gamma\left(2-N\right) + N - 1] \\ 0 & \text{elsewhere} \end{cases}. \quad (5)$$

The proposed implementation of the S-transform, given by eqn. (3), reduces to the standard S-transform for $\gamma = 0$, while for $\gamma = 0.5$ we obtain the robust median-based S-transform. In general, the proposed scheme provides us with a trade-off between spectra quality and impulse noise removal for $\gamma \in]0, 0.5[$.

III. Results and Discussion

We begin the performance analysis of the algorithm with an analysis of the following signal:

$$x_1(t) = \cos\left(\frac{800\pi}{3}(t-0.5)^3 + 30\pi t\right) + \cos\left(30\pi t + 40\pi t^2\right); \quad (6)$$

where $0 \leq t < 1$, and the assumed sampling period is $T_s = 1/256$ seconds. The signal is contaminated with additive α-stable noise. In particular, we used Cauchy noise ($\alpha = 1$) with the dispersion factor equal to 0.1.

The signal consists of two components: a linear FM component and a component with a hyperbolic frequency behavior. Using the standard algorithm, shown in Fig. 1(b), we cannot detect the components, since the noise completely masks these components in the time-frequency domain. Very similar results are obtained with the realization of the S-transform based on the median-filtered DFT as shown in Fig. 1(f). Improvements in the component localizations, i.e., the ability to detect the presence of both components, can be noticed when the S-transform is implemented with the L-DFT, as shown in Figs. 1 (c)-(e). In particular, as the value of γ descreases, we can observe a higher energy localization in the time-frequency domain. The presence of two components becomes rather obvious even in strong noise.

In order to understand the effects of the proposed S-transform implementation on the estimation of instantaneous frequency (IF), we consider the following signal:

$$x_2(t) = \sin(110\pi t + 4\pi\cos(4\pi t)) \quad (7)$$

where $x_2(t)$ is only defined in the interval given by $0 \leq t < 1$, and the sampling period is $T_s = 1/256$ seconds. The signal is contaminated with additive α-stable noise ($\alpha = 1$), whose dispersion factor is varied between 0.025 and 0.25 in steps of 0.025. The IF is estimated from the peaks of the magnitude of the time-frequency transform [6], and the mean square error (MSE) of the estimator is evaluated for the standard S-transform and the S-transform based on L-DFT with varying γ parameter ($\gamma = \{0, 0.125, 0.250, 0.375, 0.5\}$). The error is defined as a difference between the true value of the IF and the estimated value. The MSE values represent an average of 4000 realizations.

The results in Fig. 2(d) demonstrate the behavior of the instantaneous frequency estimator based on the various implementations of the S-transform. The best performance is achieved for $\gamma = 0.125$. In general, it is clear that the L-DFT based S-transform produces smaller MSE in comparison to the standard S-transform, and hence, a more accurate estimation of the instantaneous frequency. Such a behavior of the estimator is explained by TFRs of $x_2(t)$ generated by the standard S-transform and the L-DFT based transforms with $\gamma = 0.25$ and $\gamma = 0.5$, depicted in Figs.

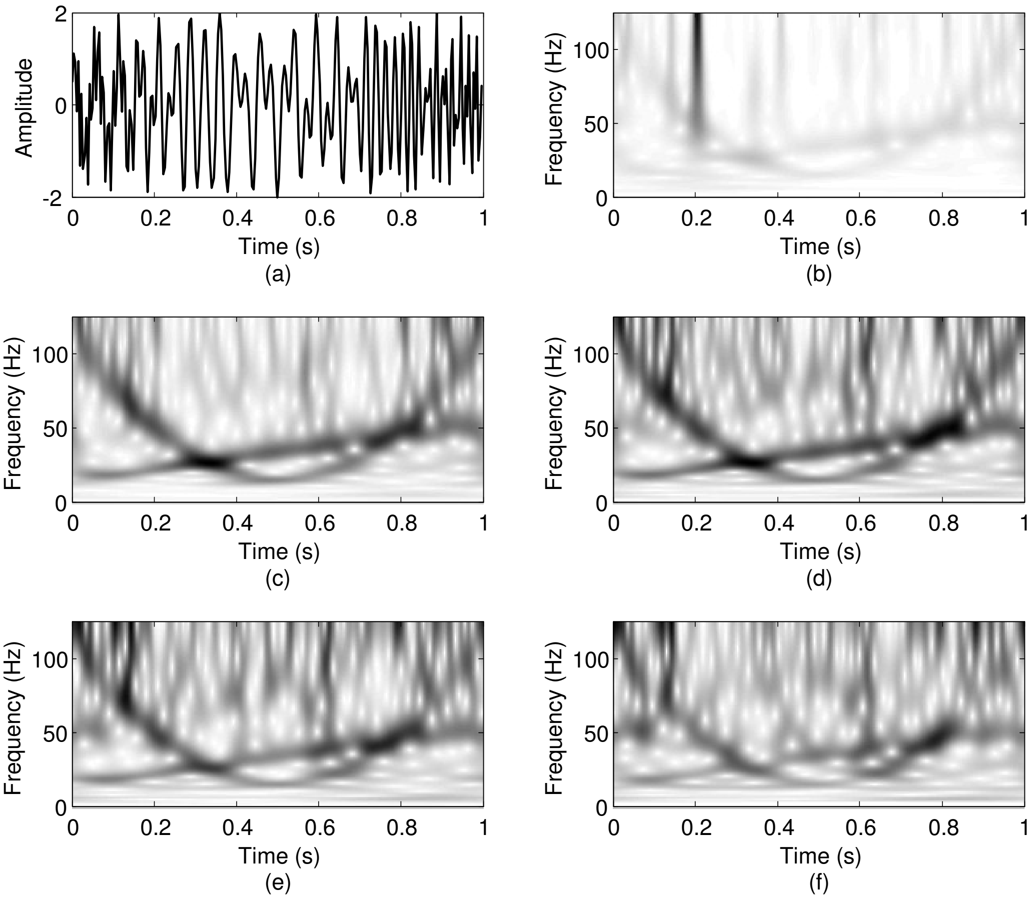

Fig. 1. Various realizations of the S-transforms: (b) standard S-transforms; and several realizations of the robust S-transform: (c) $\gamma = 0.125$; (d) $\gamma = 0.250$; (e) $\gamma = 0.375$; (f) $\gamma = 0.5$. Time-domain representation of noise-free $x_1(t)$ is depicted in (a).

2 (a)-(c), obtained for the dispersion factor equal to 0.15. From these representations, it is clear that the robust S-transform is capable of tracking the sinusoidally modulated component. On the other hand, the standard S-transform exhibits a very poor performance as shown in Fig. 2 (a).

IV. Conclusion

A novel approach for diminishing the effects of $\alpha-$stable noise on the time-frequency representations of non-stationary signals obtained by the S-transform has been proposed. The new algorithm is based on the L-DFT. The conducted numerical analysis showed that the new scheme significantly enhanced the time-frequency representations of non-stationary signals contaminated with the impulse noise. Additionally, for such signals the robust S-transform provided a more accurate approach for the instantaneous frequency estimation than the standard S-transform.

References

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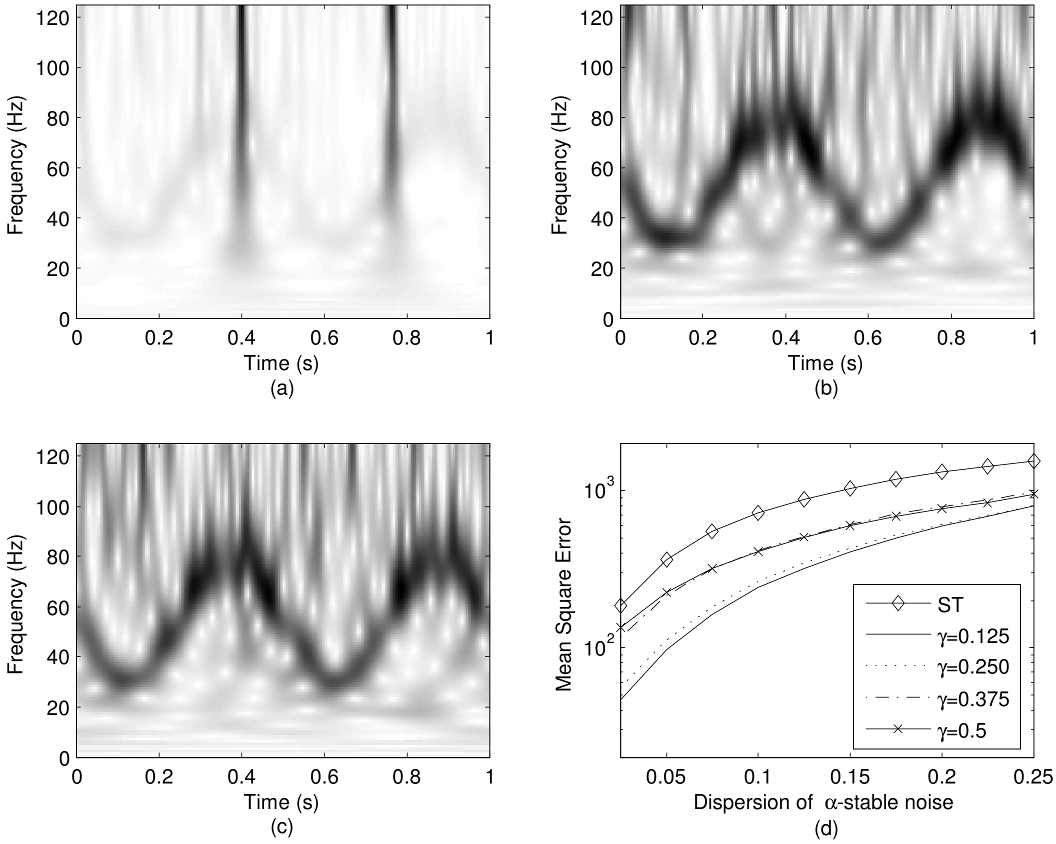

Fig. 2. Various realizations of the S-transform: (a) standard form ($\gamma = 0$); (b) L-DFT based transform ($\gamma = 0.25$); (c) median-based transform ($\gamma = 0.5$). Mean square error for instantanteneous frequency estimation (d).

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