Order Adaptive Local Polynomial FT Based Interference Rejection In Spread Spectrum Communication Systems

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Abstract— **Methods for jammer rejection in the spread spectrum communications, based on the time-frequency representations, have been proposed in order to improve the desired signal receiving performances. In this paper we consider the nonstationary jammer case. The local polynomial Fourier transform (LPFT) is used to represent the received corrupted signal. Time-varying filtering is implemented in this domain, having in mind that the LPFT is linear with respect to the signal. An order adaptive algorithm of the LPFT calculation is presented. Performance of the proposed nonparametric method is tested in the presence of linear and sinusoidal FM interferences in the noisy signal, without any a priori assumption about the jammer form. The proposed method may be successfully extended to the case of the multiple jammers. Obtained results in the terms of the bit error rate (BER) values show the achieved improvements. Since the first order LPFT is closely related to the windowed modified fractional Fourier transform, procedure for an efficient optimization is presented.**

I. INTRODUCTION

Spread spectrum is a transmission coding technique used in digital telecommunication systems, where pseudo-noise (PN), or pseudo-random code, independent of the information data, is employed as modulation waveform. This code significantly expands the bandwidth of original signal. Original purpose of this kind of modulation lied in the need for providing efficient jammer resistant communication, and low probability of intercepting. The spread signal has a lower power density, but the same total power. At the receiver side signal is "despreaded" using the synchronized replica of the pseudo-noise code. Despreading provides resistance to the interference and multipath fading. Spread spectrum technology has been recognized as a good alternative to both frequency division multiple access (FDMA) and time division multiple access (TDMA) for the cellular systems. The most common spread spectrum systems are of the direct sequence (DS) or frequency hopping (FH) type. Direct sequence SS systems employ a high-speed code sequence to introduce rapid phase transitions into the carrier containing the data. The result of modulating the carrier is a signal centered at the carrier frequency, but with main lobe bandwidth significantly wider than the original bandwidth. The FH spread spectrum achieves the band spreading by using the PN sequence to pseudo-randomly hop the carrier frequency.

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While the influence of low power interfering signals is significantly reduced by despreading process at the receiver, in case of very high power interferences preprocessing is required. This is a common case, when the interference stations are much closer to the receiver than the signal transmitting station.

Different methods have been proposed for rejection or mitigation of interferences of this kind, in order to improve interference immunity of SS systems and more reliable receiving and decoding of the useful signal. In [3] Amin proposed open-loop adaptive filtering method for the case of narrowband jammer. In [4] Wang and Amin applied multiple-zero FIR filters whose notch is in synchronization with the jammer instantaneous frequency (IF), in order to remove the jammer power at every time sample. Barbarossa and Scaglione have proposed in [5] a method based on generalized Wigner-Hough transform. They characterize linear and sinusoidal jammer by the appropriate parametric models. Suleesathira and Chaparo used a method for interference mitigation based on the evolutionary and Hough transform [6]. The fractional Fourier transform for the case of linear chirp signals can improve the presentation and overall bit error performance of the receiver when the angular parameter of the transform matches chirp rate of the interferences [7].

In this paper we propose a nonparametric approach for the jammer excision by using local polynomial Fourier transform (LPFT), which is linear with respect to the signal [8], [9]. Optimal order of the LPFT was determined and calculated for each considered instant with appropriate optimization of the selected order transform parameters. The jammer was represented in the domain of its best concentration. Time-varying filtering is then implemented in that domain. In this way the LPFT based method can be used for a general, including nonlinear FM type of interferences. The method was tested on the single linear and sinusoidal FM type of interferences and on their sum, as well. The first order LPFT may be related to the fractional FT by defining the windowed modified fractional Fourier transform (FRFT). Simple expressions for the calculation of the optimal angular parameter, based on the three fractional second order moments only, can then be used [2]. By increasing the LPFT order, removal of the higher order nonstationarities in the interference is achieved. The presented algorithm for order selection keeps the calculation complexity at a relatively low level.

Paper is organized as follows. The DS spread spectrum model was presented in Section 2. Section 3. deals with the STFT as a model of jammer filtering and signal reconstruction. In Section 4. we introduced the LPFT as a way to represent and filter jammer in optimal transform parameters domain. Section 5 presents the order adaptive algorithm and examples. Multiple jammers removal and corresponding examples are presented in Section 6.

II. DS SPREAD SPECTRUM MODEL

Let us assume a digital signal that is going to be transmitted in the waveform:

$$x(t) = \sum_n x_n h_T(t - nT) \tag{1}$$

where $x = \{x_n : x_n \in \{+1, -1\}\}$ is the data sequence and T represents data symbol duration and $h_T(t)$ is a rectangular pulse of duration T.

In the DS spread spectrum systems PN sequence may be expressed in the following manner:

$$a(t) = \sum_k a_k h_c(t - kT_c) \tag{2}$$

where $a = \{a_k : a_k \in \{+1, -1\}\}$ is a spreading sequence, T_c is the PN symbol or chip period. Pseudo-random sequence is actually a periodic deterministic sequence with period N.

It is necessary that T is an integer multiple of T_c. The ratio $G = T/T_c$, defined as a number of PN chips per data symbol, is called processing gain. There are two types of spreading codes, short code and long code. The short code is one with $G = N$, i.e. when PN code length is equal to a data symbol. The long-code has $G << N$, i.e. PN code length is much longer than the data symbol, so that a different chip pattern is associated with each symbol. The total transmitted signal may be expressed in the form:

$$s(t) = \sum_n x_n a(t - nT)$$

$$= \sum_n x_n \sum_k a_k h_c(t - kT_c - nT). \tag{3}$$

The received signal is of the following form:

$$r(t) = s(t) + J(t) + w(t), \tag{4}$$

where $s(t)$ is the desired signal, $J(t)$ is an interference and $w(t)$ is the uncorrelated white noise process with the autocorrelation function: $R_{ww}(t) = \sigma_w^2 \delta(t)$. Amount of noise in the received signal is described by the signal-to-noise ratio (SNR), defined $SNR = 20log_{10}(A/\sigma_w)$, where A is the signal amplitude. The interference (jammer) has the form:

$$J(t) = a_J \cos(\varphi(t)), \tag{5}$$

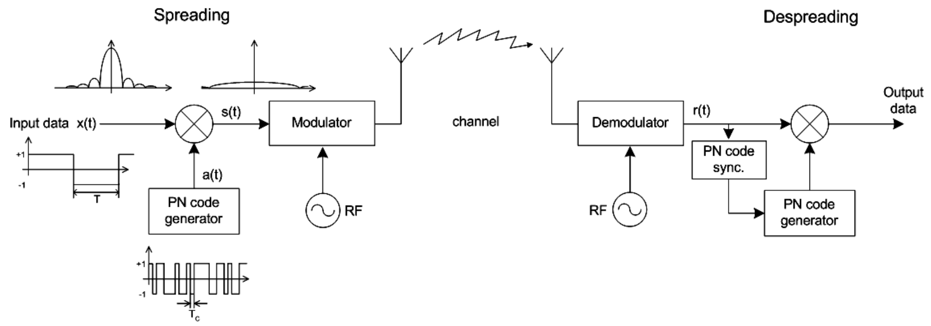

Fig. 1. Block diagram of the DS spread spectrum system. Plots above the multiplicator at the transmitter side show pdf-s of the original (before multiplication) and the spreaded signal (after multiplication).

where $\varphi(t)$ is the phase and a_J is the magnitude of the jammer. Amount of the jammer in the received signal can be described by the jammer-to-signal ratio (JSR), defined as $JSR = 10log_{10}(P_J/P_s)$, where P_J and P_s represent power of the jammer and the signal, respectively, [11], [13]. Block diagram of the DS spread spectrum (DSSS) system is given in Fig.1.

III. FILTERING AND RECONSTRUCTION MODEL

First let us use the well-known Short Time Fourier Transform (STFT) as the analysis tool for both filtering of the jammer and reconstruction of the desired signal. The STFT is defined by:

$$STFT(t,\omega) = \int_{-\infty}^{+\infty} x(t+\tau)w^*(\tau)e^{-j\omega\tau}d\tau, \quad (6)$$

where $w^*(t)$ denotes a conjugated lag-window function. One of the most important properties that STFT inherited from the FT is full reversibility. This property means that the original signal can be fully recovered from its STFT. Product of the signal and the window may be obtained by using the inverse FT, i.e.:

$$x(t+\tau)w^*(\tau) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} STFT(t,\omega)e^{j\omega\tau}d\omega. \quad (7)$$

Based on this relation and the property of the window function that $w^*(0) = 1$, it follows:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} STFT(t,\omega)d\omega. \quad (8)$$

The signal $x(t)$, at any instant, may be obtained by integrating the STFT, at the same instant, over the frequency.

The STFT is linear transform and therefore for the received signal it is composed of three components: the STFT of the desired signal, the STFT of the noise, and the STFT of the jammer. Because of spreading at the transmitter side, bandwidth of the desired signal is wider than the bandwidth of jammer. We may assume that the desired signal and noise exist over all frequencies, while the jammer exists only in a certain frequency interval of the STFT. This assumption leads to the idea of filtering jammer in the time-frequency plane. The desired signal is then reconstructed by inverting the filtered STFT. The filtering and signal reconstruction may be modeled by the following expression:

$$r_f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} L_f(t,\omega)STFT(t,\omega)d\omega. \quad (9)$$

where $r_f(t)$ is the filtered signal, $L_f(t,\omega)$ is a rectangular support function used for filtering. Analytical expression of this window is given by:

$$L_f(t,\omega) = \begin{cases} 0, & \omega \in B_j(t) \\ 1, & elsewhere \end{cases}, \quad (10)$$

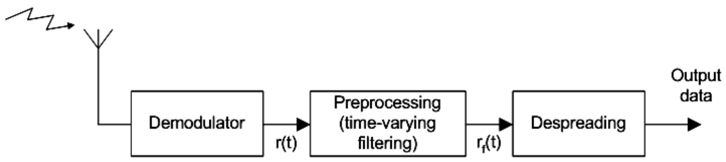

Fig. 2. Block diagram of the DSSS receiver in case of a high power jammer.

where $B_j(t)$ is the frequency band around the jammer's instantaneous frequency $\varphi'(t)$, i.e. the region in which we assume that the jammer exists. Because of its narrow frequency band it is easy to detect the jammer, even when its total power is lower or equal to the power of signal.

Block diagram of the DSSS receiver in case of a high power jammer is shown in Fig.2.

Now we will illustrate this simple and straightforward filtering forms on examples.

Example 1: Here we will demonstrate performance of the presented filtering of received signal based on the direct STFT application. Consider signal defined by (3) and (4), that has a length of 1 bit. The number of chips per bit was set to 128. The same parameters are going to be used in all further examples. Standard deviation of the added white Gaussian noise was assumed to be $\sigma_w = 2A$, where A is the amplitude of the received BPSK signal. Jammer is assumed in the form of a linear frequency modulated (FM) signal. The power of jammer was varied from 0 to 145 dB (relative to the signal's energy, JSR) in increments of 5 dB. Bit error rate (BER) values are calculated from corresponding CER (Chip Error Rate) values. We applied Kaiser and Hanning window in the STFT, both with length of 128 time samples. For Kaiser window $\beta = 15$ is used. After time-varying filtering based on (9) and (10) we reconstructed the signal, and got CER (Chip Error Rate) values. The BER values are shown in Fig. 3. The Hanning window has narrower main lobe, producing slightly better results for low JSR, while the Kaiser window has much lower side lobes, resulting in a more robust system for high JSR.

Example 2: Interference is taken as a linear-frequency modulated signal. The interference power was set to 30 dB (relative to the signal's energy). Here we vary the modulation index (slope) of the jammer in the STFT from 0 (the case that corresponds to a pure sinusoidal interference) to the value at which jammer sweeps the entire considered frequency area (the area where components of the desired signal exist). Filtering of the interference component in the received signal was done in the same way as in the previous example. The results are illustrated in Fig. 4.

Example 3: In this example we consider 2 bits of signal. Kaiser window with length 128 and beta value 15 was implemented. Interference is taken as a sinusoidal-frequency modulated signal. JSR was set to 30 dB. Here we varied the magnitude of interference frequency from 0 to the value at which jammer covers entire frequency range where the signal exist. Interference filtering was done in the same way as before. Results in BER as a function of the jammer rate are illustrated in Fig.5. As expected, when the jammer rate increases it occupies wider frequency range in the STFT, covering wider area of the desired signal, and thus increasing the BER.

IV. LOCAL POLYNOMIAL FOURIER TRANSFORM

In order to define a more robust system to the jammer variation here we will consider the Local polynomial Fourier transform (LPFT).

The LPFT has been introduced by Katkovnik as:

$$LPFT(t, \omega; \omega_1, \omega_2, ..., \omega_N) =$$

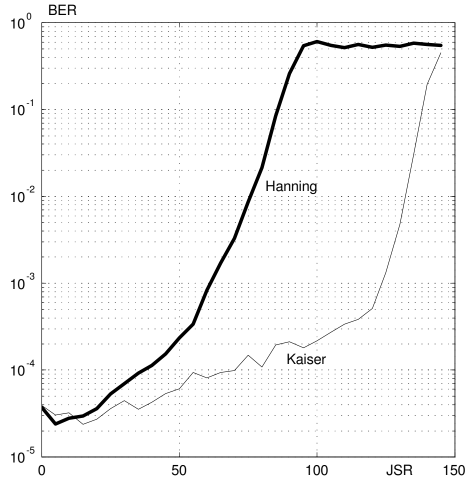

Fig. 3. Bit error rate (BER) curves: Thick line for the Hanning window; Thin lines for the Kaiser window; Windows width was 128 samples; Beta value for Kaiser window is 15.

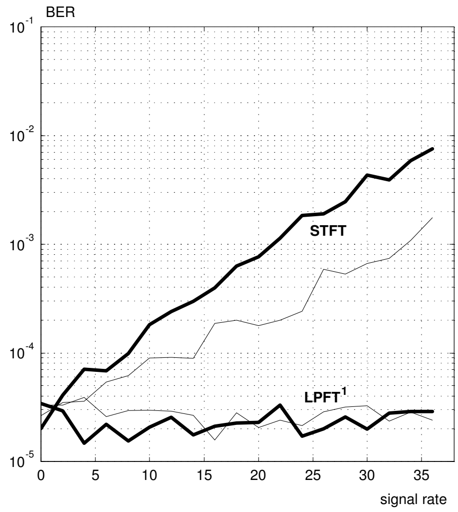

Fig. 4. Bit error rate (BER) for linear FM signal as a function of the jammer rate: Thick lines for the Hanning window; Thin lines for the Kaiser window; Upper lines for the standard and lower for the fractional based filtering. JSR=30 [dB], Windows width was 128 samples. STFT-standard short time Fourier transform based filtering. Algorithm (first order LPFT ($LPFT^1$)) - based filtering.

$$\int_{-\infty}^{+\infty} x(t+\tau)w(\tau)e^{-j(\omega\tau+\omega_1\frac{\tau^2}{2!}+\ldots+\omega_N\frac{\tau^N}{N!})}d\tau, \quad (11)$$

where $\vec{\omega} = \{\omega_1, \omega_2, ..., \omega_N\}$ represents N-dimensional parameter space. The LPFT preserves the linearity property of the STFT, with respect to the signal.

The original signal can be reconstructed from its LPFT by using:

$$x(t) = \int_{-\infty}^{+\infty} LPFT(t, \omega; \omega_1, \omega_2, ..., \omega_N)d\omega. \quad (12)$$

It is interesting to note that the integral (12) does not depend on the parameters of the N-dimensional parameter space, $\omega_1, \omega_2, ..., \omega_N$.

The optimal LPFT is the transform that

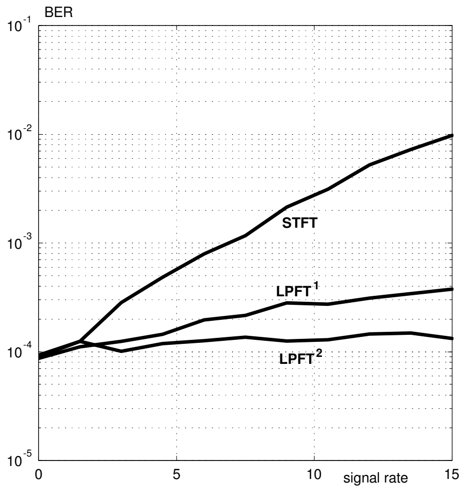

Fig. 5. Bit error rate (BER) for sinusoidal FM signal as a function of the jammer rate: Kaiser window with 128 samples is used in the implementation. JSR=30 [dB]. STFT-standard short time Fourier transform based filtering. Algorithm with the LPFTs up to the first and the second order (LPFT^1 and LPFT^2) - based filtering.

satisfies:

$$LPFT_{opt}(t,\omega) = \max_{\omega_1,\omega_2,\ldots,\omega_N}\{abs[LPFT(t,\omega;\omega_1,\omega_2,\ldots,\omega_N)]\} \tag{13}$$

Basically, optimization can be done over N-dimension parameter space. However, since the significance of the parameters decreases as their index increases, we can first optimize the LPFT with respect to ω_1, then vary ω_2 for the obtained fixed ω_1, and so on. In this way, the realization procedure may be computationally efficient.

Filtering was done in the same way as in the STFT case, i.e.:

$$x_f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} L_f(t,\omega)LPFT_{opt}(t,\omega)d\omega. \tag{14}$$

where $x_f(t)$ is the filtered received signal, $L_f(t,\omega)$ is a rectangular excision function used for filtering. Analytical expression of this window is given by:

$$L_f(t,\omega) = \begin{cases} 0, & \omega \in B_j(t) \\ 1, & elsewhere \end{cases}, \tag{15}$$

where $B_j(t)$ is the frequency band where the concentrated jammer exists.

As a special case of the LPFT, let us first consider the simplest, first order LPFT, $LPFT(t,\omega;\omega_1)$. The optimization of this transform can be done directly by varying the parameter ω_1. After the optimal (maximally concentrated) LPFT is found, the filtering relation (14) is performed. The optimization procedure may be done in a much more efficient way by exploiting the relations derived for the fractional Fourier transform [2], [12].

The first order LPFT can be related to a form of the fractional Fourier transform (FRFT), that will be referred to as the windowed modified FRFT. The FRFT of the signal $x(t)$ is defined by:

$$X_\alpha(u) = \sqrt{\frac{1-j\cot\alpha}{2\pi}}\int_{-\infty}^{+\infty} x(t)e^{j\frac{\tau^2+u^2}{2}}e^{-jut\csc\alpha}dt. \tag{16}$$

Here we will introduce the windowed FRFT:

$$WX_\alpha(t,u) = \sqrt{\frac{1-j\cot\alpha}{2\pi}}e^{j\frac{u^2}{2}\cot\alpha}$$

$$\times\int_{-\infty}^{+\infty} x(t+\tau)w^*(\tau)e^{j\frac{\tau^2}{2}\cot\alpha}e^{-jut\csc\alpha}dt. \tag{17}$$

We may denote $x(t+\tau)$ as $x_t(\tau)$ and instead of $WX_\alpha(t,u)$ we will consider its modified form

$W\tilde{X}_\alpha(t,u)$:

$$W\tilde{X}_\alpha(t,u)=\int_{-\infty}^{+\infty} x_t(\tau)w^*(\tau)e^{j\frac{\tau^2}{2}\cot\alpha}e^{-jut\csc\alpha}dt. \tag{18}$$

The relationship between $WX_\alpha(t,u)$ and $W\tilde{X}_\alpha(t,u)$ is:

$$W\tilde{X}_\alpha(t,u) = \frac{WX_\alpha(t,u)}{\sqrt{\frac{1-j\cot\alpha}{2\pi}}e^{j\frac{u^2}{2}\cot\alpha}} \tag{19}$$

Parameters in (11) and (18) are related by $\omega = u\csc\alpha$ and $\omega_1 = -\cot\alpha$. Optimal fractional domain can be found by maximizing the concentration of $W\tilde{X}_\alpha(t,u)$ by varying the parameter α (angle of rotation). Alternative way for obtaining optimal α value, i.e. parameters in the first order LPFT, is based on the results presented in [1], [12]. The optimal α value can analytically be determined as:

$$\alpha_{opt} = \frac{1}{2}\arctan\left(-\frac{m_0 + m_{\pi/2} - 2m_{\pi/4}}{m_0 - m_{\pi/2}}\right), \tag{20}$$

where m_0, $m_{\pi/2}$ and $m_{\pi/4}$ are (normalized and centered) second order moments for $\alpha = 0$, $\alpha = \pi/2$ and $\alpha = \pi/4$, respectively:

$$m_\alpha = \frac{1}{E}\int_{-\infty}^{+\infty} |F_\alpha(x)|^2 x^2 dx, \tag{21}$$

where $F_\alpha(x)$ represents the fractional FT, and E represents the zero order moment (the energy of the signal):

$$E = \int_{-\infty}^{+\infty} |F_\alpha(x)|^2 dx. \tag{22}$$

Notice that the optimal value for α can be calculated if we know second order moments of the fractional FT for three angle values only. After α_{opt} is calculated, we are able to filter received signal in new domain, with optimally concentrated jammer.

Now we are going to describe algorithm of adaptive interference rejection.

V. ORDER ADAPTIVE ALGORITHM

Within this section we will introduce and present an order adaptive algorithm that will provide additional savings in computation. It is based on the following property of the LPFT. Consider definition (11). It is easy to conclude that when we achieve that the jammer form and the LPFT order are matched within the window $w(t)$ then:

$$LPFT(t,\omega;\omega_1,\omega_2,...,\omega_N) = a_J W(\omega - \varphi'(t)). \tag{23}$$

Thus, the LPFT order is matched to the jammer form when the filtering region $L_f(t,\omega)$, for a given t, is approximately equal to the width of the window's main lobe.

Take initial time instant of the received signal into consideration.

Step 1. Calculate $STFT(t,\omega)$ for a given instant t and determine the filtering region $L_f(t,\omega)$ (frequency range) covered by the jammer for that instant.

If the width of $L_f(t,\omega)$ is approximately equal to the width of the main lobe of applied window, it means that the jammer frequency is constant within the considered time interval (width of the lag window). Filtering is done according to relation (14). Go to step 4.

Otherwise go to the next step.

Step 2. Calculate the first order LPFT and perform its optimization (maximizing of concentration) by varying the parameter ω_1. Optimization of the first order LPFT can be done in an efficient manner by maximizing the concentration of $W\tilde{X}(t,u)$ exploiting relation (20). If the width of such a determined $L_f(t,\omega)$, in the optimal ω_1 domain (for $\alpha = \alpha_{opt}$), is approximately equal to the width of the main lobe of applied window, the filtering is realized based on relation (14). Go to step 4. Otherwise go to the next step.

Step 3. Calculate the second order LPFT and perform its optimization by varying the parameter ω_2. Apply filtering, like in the previous steps, and go to step 4. We may proceed with this algorithm, for third, fourth..., order LPFT, but we will restrict ourselves to the second order LPFT. The reason for that is in fact that the results obtained for up to the second order LPFT are almost independent of the jammer rate.

Step 4. Take a next time instant t, and go to step 1.

In the next two examples we are going to demonstrate contribution in the jammer excision provided by the described algorithm. As it can be seen from Fig. 6, this contribution is reflected in "narrowing" of the area corrupted by the jammer.

Example 4: Consider the received signal composed of the same components as in Example 2. Now, in addition to the standard STFT we will use the presented algorithm. The jammer parameters are the same as in Example 2. Since the jammer is a linear FM signal the algorithm used the first order LPFT. After calculating the filtering range and interference filtration based on (14) and (15), we got results for BER illustrated in Fig. 4.

Example 5: Now consider received signal composed of the same components as in Example 3 and apply the order adaptive LPFT. Two forms of the algorithm are used. In the first case we limited the algorithm up to the first order LPFT (represented by LPFT^1). In the second case a full form of the presented algorithm was used (up to the second order LPFT (represented by LPFT^2). In this way we obtained the filtered signal and BER curves denoted by LPFT^1 and LPFT^2 in Fig. 3. The algorithm, up to the first order LPFT, improved significantly BER with respect to one obtained by using the STFT. Further improvement is achieved by using the algorithm up to the second order LPFT. As we can see the algorithm here produces results almost independent of the signal rate (amplitude of the jammer frequency variations). It means that there is no need for increasing the LPFT order in the algorithm, in this case.

Illustration of the filtering region for one realization and all three used filtering forms (STFT, LPFT^1 and LPFT^2) is given in Fig.6. Filtered out area in the time frequency plane is shown in white. Transforms are calculated for 2 bits of signal with 128 chips in each bit. Sinusoidal jammer with JSR=30 dB is considered. From this figure we see improvement in localization when the algorithm uses higher order LPFTs. It results in the performance improvement.

VI. ORDER ADAPTIVE ALGORITHM FOR MULTIPLE JAMMERS REMOVAL

Proposed algorithm may be extended to the case of the multiple jammers. Now we treat interference as a sum of P components:

$$J(t) = \sum_{k=1}^{p} a_k \cos(\varphi_k(t)), \qquad (24)$$

where $\varphi_k(t)$ is the phase and a_k is the magnitude of the k-th jammer.

Filtering of the k-th jammer can be done in the same way as in (14), i.e.:

$$x_f^k(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} L_f^k(t,\omega) LPFT_{opt}^k(t,\omega) d\omega$$

$$L_f^k(t,\omega) = \begin{cases} 0, & \omega \in B_k(t), k = 1,2,...,P \\ 1, & elsewhere \end{cases}, \qquad (25)$$

where $x_k^f(t)$ is the received signal with k-th jammer filtered out, $L_f^k(t,\omega)$ is a rectangular excision function used for filtering of the k-th jammer, and the $LPFT_f^k(t,\omega)$ is LPFT of the received signal with k-th jammer optimally concentrated. Analytical expression of the excision function is of the form (25), where $B^k(t)$ is the frequency band where the optimally concentrated k-th jammer exists.

Algorithm for the multiple jammers removal is based on the algorithm for the single jammer removal. The only difference lies in the need for repeating steps 1, 2 and 3 in succession, until the all jammer components are completelly removed.

Take the initial time instant of the received signal into consideration.

Step 1. Calculate $STFT(t,\omega)$ for a given instant t. Detect the existence of a very strong jammer and the number of its components (high peaks in the STFT for a given instant). If $STFT(t,\omega)$ does not contain interferences go to step 4. Otherwise, adapt the signal transform to the strongest jammer component. To this aim form an auxiliary spectrum that contains only one jammer by setting all frequency components "corrupted" by the other

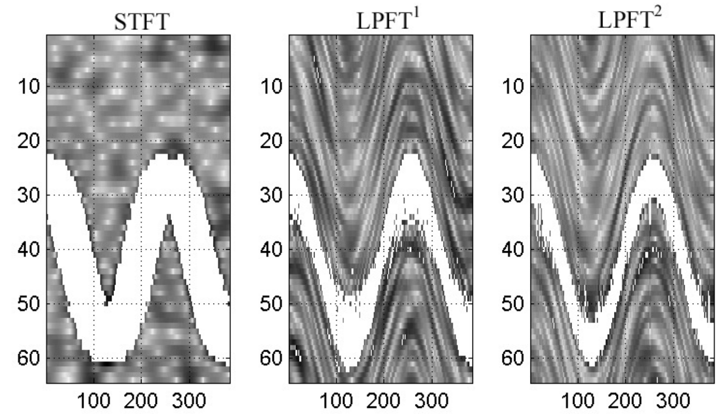

Fig. 6. Absolute values of the transforms: 1) STFT, 2) transform obtained with the algorithm up to the first order LPFT ($LPFT^1$), and 3) transform obtained with the algorithm up to the second order LPFT ($LPFT^2$) with corresponding filtered out areas for one realization of the received signal and sinusoidally modulated FM jammer. Horizontal axis - time, Vertical axis -frequency.

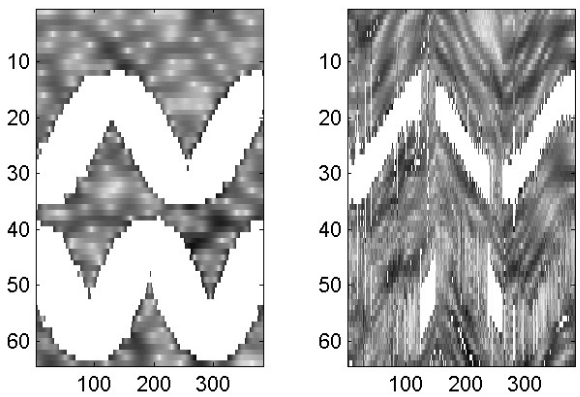

Fig. 7. Absolute values of the transforms: 1) STFT, 2) LPFT, with corresponding filtered out areas for one realization of the received signal and sum of the two sinusoidally modulated FM jammers. Horizontal axis - time, Vertical axis - frequency.

jammers to zero, and form the auxiliary signal as inverse FT of the auxiliary spectrum. Determine the frequency range covered by the jammer left in the auxiliary signal for that instant. If that range is approximately equal to the width of the main lobe of applied window (the jammer is pure sinusoid) perform filtering of the original signal according to relation (25). Go to step 1 with this filtered signal.

Otherwise, go to the next step.

Step 2. Calculate the first order LPFT of the auxiliary signal and perform its optimization. If the width of the concentrated jammer, in the optimal ω_1 domain, is approximately equal to the width of the main lobe of applied window (the jammer is linear FM signal) calculate the first order LPFT of the original signal with $\omega_1 = \alpha_{opt}$ and apply filtering based on relation (25). Go to step 1 with this filtered signal. Otherwise, go to the next step.

Step 3. Calculate the second order LPFT of the auxiliary signal and perform its optimization by varying the parameter ω_2. Calculate the second order LPFT of the original signal

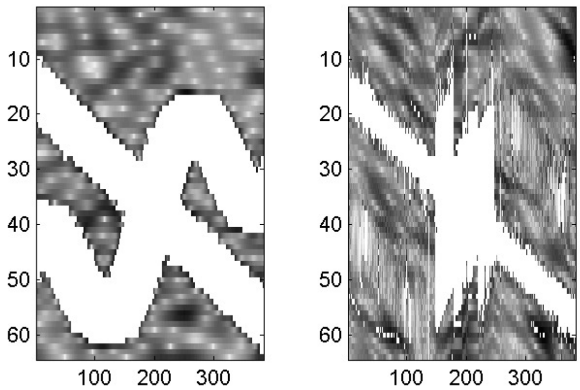

Fig. 8. Absolute values of the transforms: 1) STFT, 2) LPFT, with corresponding filtered out areas for one realization of the received signal and sum of one linear frequency modulated jammer and one jammer modulated by a sum of linear function and a sinusoid.

for obtained ω_{2opt} and ω_{1opt} and apply filtering like in the previous steps, and go to step 1. We will not proceed with this algorithm for higher order LPFTs because of the reason given in the third step of the first algorithm.

Step 4. The original signal at considered time instant contains no jammers, i.e. relation (25) previously used corresponds to the removal of the last jammer in the spectrum and the signal reconstruction, for that instant. Take a next time instant t, and go to step 1.

The next two examples demonstrate performances of the described algorithm in the multiple jammers excision.

Example 6:. Instead of one jammer we now consider a case of two jammers, which are taken as sinusoidal frequency modulated signals. Parameters of the signal and noise are the same as in the first example. Jammers are assumed not to intersect in the time-frequency plane. After performing 50 runs of the STFT and LPFT based filtrations and signal reconstructions, we got results for BER: $1.52\text{x}10^{-2}$ for the STFT and $2.69\text{x}10^{-3}$ for the LPFT. Illustrations of the filtering regions for one realization of these filtering forms are given in Fig.7.

Example 7:. In this example as interference we take sum of one linear frequency modulated jammer and one jammer modulated by a sum of linear function and a sinusoid. Frequency ranges of the jammers are chosen in order to sweep the entire frequency area of the signal. Now we assume that the jammers intersect in the time-frequency plane. After performing 50 runs of the STFT and LPFT based filtrations and signal reconstructions, we got results for BER: $2.18\text{x}10^{-2}$ for STFT and $3.12\text{x}10^{-3}$ for LPFT. Illustrations of the filtering regions for one realization of these filtering forms are given in Fig.8. As it was expected, the jammer concentration is not achieved in the area of intersection of the jammers.

VII. CONCLUSION

Local polynomial Fourier transform is used to represent and filter jammer in communication systems. Time-varying filtering was implemented in the optimal transform parameters domain. This nonparametric approach is tested on the linear and sinusoidal FM signals with the algorithms using the first and the second order LPFTs. Proposed method is extended to the case of the multiple jammers. Bit error performances are significantly improved with respect to the standard STFT based systems.

VIII. ACKNOWLEDGEMENT

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