A Relationship between the Robust Statistics Theory and Sparse Compressive Sensed Signals Reconstruction

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Abstract

An analysis of robust estimation theory in the light of sparse signals reconstruction is considered. This approach is motivated by compressive sensing concept which aims to recover a complete signal from its randomly chosen, small set of samples. In order to recover missing samples, we define a new reconstruction algorithm. It is based on the property that the sum of generalized deviations of estimation errors, obtained from robust transform formulations, has different behaviour at signal and non-signal frequencies. Additionally, this algorithm establishes a connection between the robust estimation theory and compressive sensing. The effectiveness of the proposed approach is demonstrated on examples.

I. Introduction

In the last few years compressive sensing has been an emerging research area that has attracted the attention of many researchers [1]-[6]. The reason can be found in the fact that many applications, based on the Nyquist sampling rate, require a huge amount of samples (measurements). Consequently, the signal acquisition increases demands for hardware resources and very often makes real-time applications complex and expensive. The concept of compressive sensing or compressive sampling has been considered [7]-[13], in order to reduce the number of samples that are necessary for efficient signal analysis and reconstruction. The algorithms for compressive sensing (CS) deal with an incomplete, small set of signal samples (measurements or observations). If the signal satisfies sparsity property the entire data set can be fully recovered by using powerful iterative optimization algorithms. Namely, signal sparsity in one and density in other domain are crucial conditions that should be satisfied, to ensure a successful application of CS algorithms. Sparsity means that a number of non-zero coefficients in certain domain (discrete Fourier - DFT, Wavelet domain, local polynomial DFT, or any other transform domain) should be smaller than a number of available signal samples. Some of the commonly used algorithms for sparse signal reconstruction are: Primal dual interior point method, Orthogonal matching pursuit and Block orthogonal matching pursuit algorithm [6]. These algorithms are based on iterative procedures which can be time consuming and computationally demanding. The aim of this paper is to establish and analyze a relationship between robust statistics [14]-[16], used in signal estimation theory, and the compressive sensing. It leads to an algorithm for signal reconstruction, which does not always require iterative implementation, thus relaxing the algorithm complexity. Moreover, the algorithm offers flexibility of using different types of minimization norms. In this case, the sparsity of the signal in the Fourier domain and a small number of randomly positioned samples are assumed as well. The considered relationship between the two concepts is based on the initial robust formulations of the signal transforms [13], [14], and the property that incomplete set of samples causes random deviations of the DFT outside the signal frequencies. The DFT values at the frequencies corresponding to signal components are characterized by non-random behavior. Thus, the sum of generalized deviations of the values at non-signal frequencies is constant and higher than at the signal components positions. Hence, the problem of signal reconstruction is reduced to the detection and determination of a reference level of generalized deviation sum.

The paper is organized as follows. The theoretical framework of the sparse signal reconstruction is given in Section II together with the analysis of errors in the sense of robust estimation formulations in the DFT domain. The general algorithm that relates the robust formulations and compressive sensing for sparse signal reconstruction is described in Section III. The experimental results are provided in Section IV, while the concluding remarks are given in Section V.

II. Theoretical Framework

Consider a signal that consists of *K* sinusoidal components in the form:

$$x(n) = \sum_{i=1}^{K} A_i \exp(j2\pi k_{0i} n/N + \varphi_i), \tag{1}$$

where k_{0i} denotes the *i*-th signal frequency, while A_i and φ_i denote the amplitude and phase of the *i*-th component, respectively. Therefore, we might say that the signal $x(n)$ is $K-$sparse in the DFT domain. Namely, the signal sparsity in the frequency

domain is defined as follows: the number of non-zero coefficients within the Fourier transform is smaller than the number of available signal samples. This means that the signal $x(n)$ can be represented using its K-sparse DFT representation:

$$\mathbf{x} = \mathbf{\Psi X},$$

where $\mathbf{\Psi}$, $\mathbf{X}$ and $\mathbf{x}$ are DFT matrix, the vector of DFT coefficients and the signal vector, respectively. Furthermore, we assume that instead of the whole set of *N* signal samples, we deal with only $M \ll N$ samples. The available samples are usually referred to as measurements. The *M* measurements are randomly chosen from the original signal **x**, using the random measurement matrix $\mathbf{\Phi}$ of size $M \times N$:

$$\mathbf{y} = \mathbf{\Phi x}.$$

According to the compressive sensing theory the entire signal of interest **x** or equivalently its DFT vector **X,** can be reconstructed from an incomplete set of measurements **y**, with a high probability, by solving a convex optimization problem. This problem of reconstruction has the following form [1]-[4]:

$$\min \|\mathbf{X}\|_{\ell_1} \quad \text{subject to} \quad \mathbf{y} = \mathbf{\Phi\Psi X}, \tag{2}$$

where ℓ_1 minimization norm is considered. If the matrix $\mathbf{A_{cs}}$=$\mathbf{\Phi\Psi}$ is incoherent, the problem can be solved by using linear programming. After a number of iterations the optimal solution is achieved.

In the sequel we will show that a class of these problems can be solved using an approach based on the generalized deviation calculation (note that the flexibility of norm selection makes this algorithm more powerful for a wide class of signals and applications). In that sense, inspired by the formulations of the robust transform domain representations (based on the minimization procedures) [15], we may observe a loss function in the form:

$$F\{e(n)\} = F\left\{\left|x(n)\exp^{-j2\pi kn/N} - X(k)\right|\right\}, \tag{3}$$

where $F\{\}$ is a function corresponding to a certain norm. The total error, over all time values, is:

$$I = \sum_{n=0}^{N-1} F\left\{\left|x(n)\exp^{-j2\pi kn/N} - X(k)\right|\right\}. \tag{4}$$

Note that the standard definitions of the transform domain representations correspond to l_2 norm, i.e. to the squared error: $F\{e(n)\} = |e|^2$, while the norm l_1 is obtained with $F\{e(n)\} = |e|$. They are obtained by minimizing the total error.

III. Estimation of Generalized Deviation

In the case of compressive sampled signal $x(n)$, the number of available samples is much fewer than N. For instance, for a considered signal with $K > 1$ frequency components, denote the number of available samples by *M*, where $M > K$ holds. Thus, the samples are chosen randomly in *M* time points defined by the set $N_{avail} = \{n_1, n_2, ..., n_M\}$:

$$x(n_1), x(n_2), x(n_3), ..., x(n_M). \tag{5}$$

The errors for each of them are:

$$\begin{gathered} e(n_m, k) = \left|x(n_m)\exp^{-j2\pi kn_m/N} - X(k)\right| \\ n_m \in N_{avail} = \{n_1, n_2, ..., n_M\} \end{gathered} \tag{6}$$

where $X(k)$, $k = 0, 1, 2, ..., N$ is the Fourier transform of the signal $x(n)$. For the standard forms with $F\{e(n)\} = |e|^2$, the estimated DFT is:

$$\hat{X}(k) = \underset{n_m \in N_{avail}}{\text{mean}} \{x(n_1)\exp^{-j2\pi kn_1/N}, ..., x(n_M)\exp^{-j2\pi kn_M/N}\}. \tag{7}$$

It represents the maximum likelihood (ML) estimate for full set of data and Gaussian noise. The robust estimate is obtained using $F\{e(n)\} = |e|$. It is given by:

$$\hat{X}(k) = \underset{n_m \in N_{avail}}{\text{median}} \{x(n_1)\exp^{-j2\pi kn_1/N}, ..., x(n_M)\exp^{-j2\pi kn_M/N}\}. \tag{8}$$

Since the data set is complex-valued, the marginal median could be used, instead of the complex (vector) median form. The marginal median is the median applied independently to the real and imaginary parts of the set values. This robust form may significantly improve results over the standard estimates, especially in the case when an incomplete set of noisy data is analyzed.

After calculating the error values for each available sample, based on the loss function $F\{\}$, we can calculate the sum of general deviations for each frequency $k = 0, 1, ..., N$:

$$GD(k) = \frac{1}{M} \sum_{n_i \in N_{avail}} F\{|e(n_i, k) - \text{mean}\{e(n_1, k), e(n_2, k), ..., e(n_M, k)|\} = \tag{9}$$

$$= \text{dev}\{e(n_1, k), e(n_2, k), ..., e(n_M, k)\}, \tag{10}$$

where the generalized loss function can be written in the form: $F\{e\} = |e|^L$. The errors based on the l_1 norm are obtained for $L = 1$, while the l_2 norm based errors are obtained for $L = 2$. It is important to emphasize that we have a flexibility to use different norms by changing only the value of parameter L. In the case when the loss function is $F\{e\} = |e|^2$, the variance is obtained:

$$GD(k) = V(k) = \text{var}\{e(n_1, k), e(n_2, k), ..., e(n_M, k)\}, \quad \text{for } k = 1, 2, ..., N. \tag{11}$$

Depending whether the frequency points correspond to the position of signal components in the Fourier transform domain or not, we may observe the following two cases:

1) For a frequency point corresponding to the *p*-th signal component, we have:

$$e(n_m, k) = \left| \sum_{i=1, i \neq p}^{K} A_i \exp(j2\pi(k_{0i} - k)n_m/N) \right| \tag{12}$$

for $n_m \in N_{avail} = \{n_1, n_2, ..., n_M\}$.

In the case of $L = 2$, for a given $k = k_{0p}$, we can write:

$$GD(k_{0p}) = \text{var}\{e(n_1, k_{0p}), e(n_2, k_{0p}), ..., e(n_M, k_{0p})\} = \frac{M(N-M)}{N-1} \sum_{i=1, i \neq p}^{K} A_i^L. \tag{13}$$

Therefore,

$$\text{var}\{e(n_1, k_{0p}), e(n_2, k_{0p}), ..., e(n_M, k_{0p})\} = \frac{M(N-M)}{N-1} \sum_{i=1, i \neq p}^{K} A_i^L < \frac{M(N-M)}{N-1} \sum_{i=1}^{K} A_i^L \tag{14}$$

is used as an indicator that the current frequency $k = k_{0p}$ belongs to the signal component, and the signal component is obtained as follows:

$$X(k_{0p}) = \frac{1}{M} \sum_{m=1}^{M} x(n) \exp(-j2\pi k_{0p} n_m/N). \tag{15}$$

2) Now we have to show that the sum of generalized deviations calculated for frequencies where signal components are not present is different comparing to the previous case. Hence, for $k = kq \neq k_{0i}$, $i = 1, 2, ..., K$, i.e., when the observed *k* does not belong to any signal component, we have:

$$e(n_m, k) = |\sum\nolimits_{i=1}^{K} A_i \exp(j2\pi(k_{0i} - k)n_m/N)|, \tag{16}$$

where $n_m \in N_{avail} = \{n_1, n_2, ..., n_M\}$. Then obviously, for any $k = k_q \neq k_{0i}$, $i = 1, 2, ..., K$ we can conclude that $e(n_1, k_q) \neq e(n_2, k_q) \neq ... \neq e(n_M, k_q)$, while the variance is obtained as:

$$GD(k_q) = \text{var}\{e(n_1, k_q), e(n_2, k_q), ..., e(n_M, k_q)\} = \frac{M(N-M)}{N-1} \sum_{i=1}^{K} A_i^L = const. \tag{17}$$

In the case of $L = 2$, the variances of random error at $k = k_{0i}$ and $k \neq k_{0i}$, $i = 1, 2, ..., K$ are asymptotically related as

$$\sum_{i=1, i \neq p}^{K} A_i^2 / \sum_{i=1}^{K} A_i^2 < 1.$$

The above relation holds for the remaining $(N-K)$ frequencies that do not correspond to signal components. The corresponding DFT values for these non-signal samples should be set as:

$$X(k_q) = 0. \tag{18}$$

For the sake of simplicity and clarity of explanations, but without loss of generality, in the sequel we present more details for the case *L*=2, i.e., variance estimation based procedure. However, we will also show efficiency and advantages of using ℓ_1 norm instead of ℓ_2 norm.

IV. Algorithm for CS Reconstruction of Sparse Signals

In order to summarize and clarify previously presented theory, first we present the variance based iterative algorithm for sparse signals reconstruction. The steps of the algorithm can be described as follows:

1. For each $k = 0, 1, ..., N$ calculate $\hat{X}(k)$ and the generalized deviation $GD(k)$. For norm l_2 use $F\{e(n)\} = |e|^2$ and (7), while for norm l_1 use $F\{e(n)\} = |e|$ and (8).

2. Determine the position of the first minimum in order to remove the highest component A_1:

$$k_{01} = \arg\min\{GD(k)\}, \ for \ k = 1, ..., N. \tag{19}$$

3. After localizing the first sinusoid, we estimate the value of $X(k_{01})$ as:

$$X(k_{01}) = mean\{x(n_1)\exp^{-j2\pi k_{01}n_1/N}, x(n_2)\exp^{-j2\pi k_{01}n_2/N}, ..., x(n_M)\exp^{-j2\pi k_{01}n_M/N}\}, \tag{20}$$

and remove it from the set of available samples by using the following set of equations:

$$\begin{aligned} x(n_1) &= x(n_1) - X(k_{01})\exp^{j2\pi k_{01}n_1/N} \\ &... \\ x(n_M) &= x(n_M) - X(k_{01})\exp^{j2\pi k_{01}n_M/N} \end{aligned} \tag{21}$$

4. Continue in the same way to detect and eliminate the next signal component A_2 that appears at k_{02}, until the last component is localized.

5. Set $X(k) = 0$ for $k \neq k_{0i}, i = 1, 2, ..., K$. Otherwise, for $k = k_{0i}$, the exact DFT values can be calculated by solving the set of equations, for the localized frequencies, as follows:

$$\sum_{i=1}^{K} X(k_{0i})e^{j2\pi k_{0i}n_m} = x(n_m). \tag{22}$$

This is a basic form of the algorithm that directly follows from the previously presented theory, which assumes the number of iterations corresponding to the expected components number. The number of components and the number of iterations might be known from the nature of physical processes appearing in real application. On the other hand, if the number of components is not known a priori, then similarly as in other iterative approaches (e.g., OMP), the stopping criterion can be determined using the ℓ_2 norm bounded residue that remains after removing previously detected components. In other words, the procedure may end when the energy of the residual is small enough.

However, for various signals, this procedure can be recast as a non-iterative one. After calculating the vector of generalized deviations (or in the specific case the variance vector $V(k)$), we can determine the signal support as follows:

$$k_{0i} = \arg\min\{V(k) < T\}, \ for \ k = 1, ..., N, \tag{23}$$

where T represents a certain threshold which can be calculated with respect to the max$\{V(k)\}$, e.g. αmax$\{V(k)\}$ (α is a constant between 0.85 and 0.95). The parameter alpha is set heuristically, using the starting assumption that the threshold needs to select positions of minimal deviations corresponding to desired frequencies (e.g., positions having values bellow 85% of maximum).

In both cases, the CS matrix is formed starting from the DFT matrix, from which we keep only the columns corresponding to the available measurements $n_m \in N_{avail} = \{n_1, n_2, ..., n_M\}$ and the rows that correspond to the extracted frequencies k_{0i}. Since there are more equations than unknowns, the system $\mathbf{A_{cs}X} = \mathbf{y}$, or $\mathbf{XA_{cs}^T} = \mathbf{y}^T$, is solved in the least square sense, by using MATLAB operation or by using pseudo inversion:

$$\mathbf{X} = \mathbf{y}^T/\mathbf{A_{cs}^T} \ \ or \ \ \mathbf{X} = \left(\mathbf{A_{cs}^*A_{cs}}\right)^{-1}\mathbf{A_{cs}^*y}. \tag{24}$$

The reconstructed amplitudes A_i of coefficients in X, containing initial phases φ_i, are exact, for all frequencies k_{0i}.

V. Experimental Results

*Example 1:*In this first example, we aim to show the advantage of using the proposed approach. Thus, we consider a multicomponent signal consisted of sinusoidal components with very close values of amplitudes:

$$x(n) = 1.1\exp(j2\pi 16n/N) + \exp(j2\pi 32n/N) + 0.95\exp(j2\pi 64n/N).$$

Since these components behave in a similar manner, the intention is to estimate all components at once, without using the iterative procedure, from a small number of random signal measurements (32 samples which is 25% of the total signal length N=128). For that purpose, after calculating the variance vector $V(k)$ for frequency points k=1,...,N, we introduce a simple

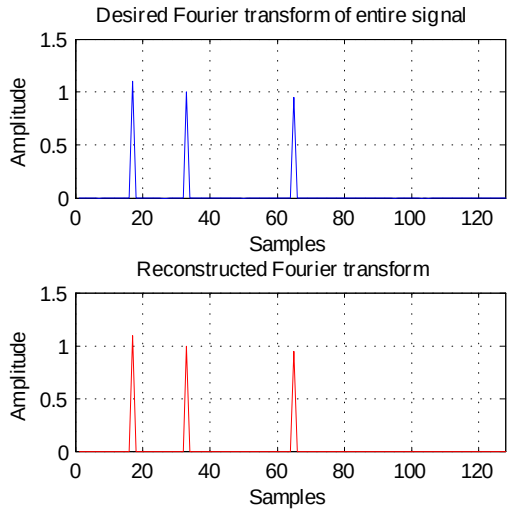

Fig. 1. Original and reconstructed Fourier transform

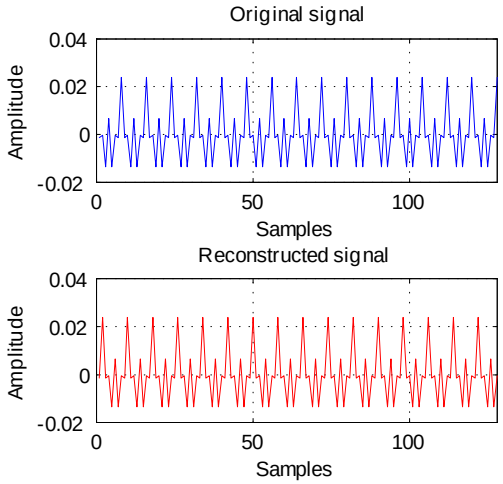

Fig. 2. Original and reconstructed signal

threshold condition to select a number of local minimum points, where this number should correspond to the number of signal components that we need to estimate. A simple threshold condition can be defined as follows:

$$\overrightarrow{k} = \arg\{V < 0.85 \max\{V\}\}.$$

Hence, vector $\overrightarrow{k}$ will contain the frequency positions of desired signal components: $\overrightarrow{k} = (k_{01}, k_{02}, k_{03})$. As described in the previous Section, the frequency components are obtained as a solution of the following system of equation: $\mathbf{X} = (\mathbf{A}^*_{\mathbf{cs}}\mathbf{A}_{\mathbf{cs}})^{-1}\mathbf{A}^*_{\mathbf{cs}}\mathbf{x}(N_{avail})$, where $\mathbf{x}(N_{avail})$ denotes a vector of signal measurements of length 32 samples, while $\mathbf{A}_{\mathbf{cs}}$ is the CS Fourier transform matrix with 32 rows corresponding to the time measurements and 3 columns corresponding to the detected signal frequency points. The achieved results are presented in Fig.1 and Fig.2. The estimated amplitudes of signals components are exactly 1.1, 1 and 0.95.

Example 2: A sparse signal is composed of seven sinusoidal components:

$$x(n) = \sum_{i=1}^{7} A_i \exp(j2\pi f_i n/N),$$

where **A**=[3, 2, 2, 6, 4, 2, 2] and **f**=[8, 15, 32, 64, 80, 96, 112], while the total number of signal samples is $N = 128$. We have randomly selected 25% of signal samples, that are used for the iterative signal reconstruction. The Fourier transform of the original signal with the complete set of 128 samples is shown in Fig. 3 (top), while Fig. 3. (bottom) shows the Fourier transform recovered from the 25% observations. The time domain representations of the original and reconstructed signal are

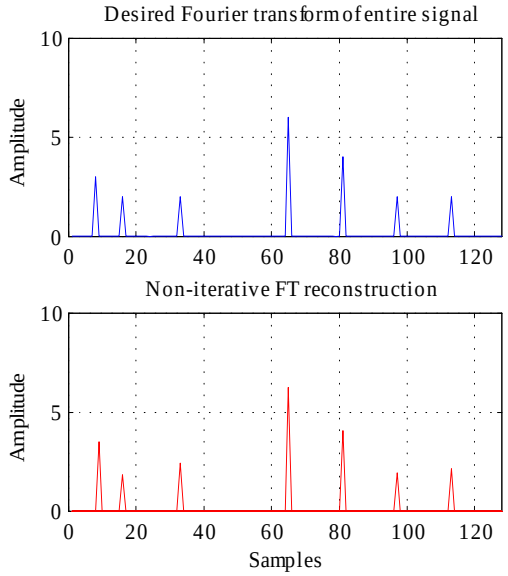

Fig. 3. Fourier transform of the original and reconstructed signals

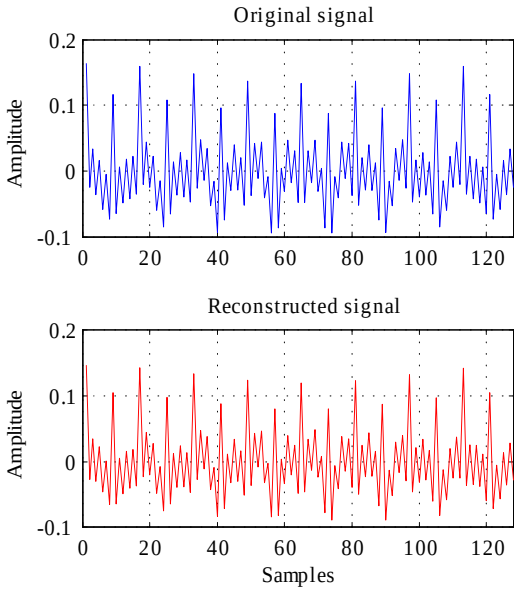

Fig. 4. Original signal with complete set of samples, recovered signal from incomplete set of samples

shown in Fig. 4. The mean square error (mse) is calculated between the original and reconstructed signal for 50 realizations (combinations of 25% of random observations) in the time domain and compared to the mean amplitude of signal samples. The mean value of mse in different realizations is 3.5% of the mean signal amplitude.

Example 3: Now, let us consider the case of non-iterative procedure applied to the signal components that differ in amplitudes as follows:

$$x(n) = 4\exp(j2\pi 16n/N) + 3\exp(j2\pi 32n/N) + 2\exp(j2\pi 64n/N).$$

The set of measurements consists of 30% of randomly chosen signal samples, out of *N*=128. The threshold condition is modified as follows:

$$\overrightarrow{k} = \arg\{V < 0.98 median\{DG\}\},$$

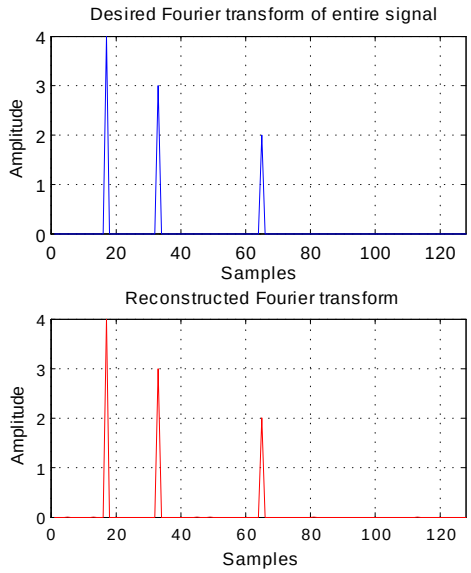

Fig. 5. Original and reconstructed Fourier transform

while the *GD* ís considered in the form of errors based on the ℓ_1 norm. In this case, the vector $\overrightarrow{k}$ might suggest slightly higher number of components than exist within the signal, but it does not affect the precision of reconstruction. Namely, instead of 3, we can obtain 7 values within $\overrightarrow{k}$ which will be used in the optimization problem: $\mathbf{X} = \left(\mathbf{A}_{\mathbf{cs}}^{*}\mathbf{A}_{\mathbf{cs}}\right)^{-1}\mathbf{A}_{\mathbf{cs}}^{*}\mathbf{x}(N_{avail})$, but in the resulting Fourier transform **X**, the insignificant components will be zero valued. The original and reconstructed Fourier transform are shown in Fig. 5.

Example 4: Here, we consider an example with the signal corrupted by impulse noise. The signal consists of three components with different amplitudes as in the previous example, while additive noise contains 2 noisy pulses (Fig. 6). The reconstruction is done as follows. The generalized deviations are calculated using the forms corresponding the ℓ_1 and ℓ_2 norms. Instead of using the thresholds, we rather assume that the number of components that we are looking for are known, and thus, we determine three minima in the generalized deviation vectors V_1 and V_2. The number of time domain observation is 64 (out of 128), i.e., we use 50% of the original signal, while making sure that we have included both pulses within the measurements. The reconstructed versions of signal and its Fourier transform are shown in Figs. 6 and 7, respectively.

It is obvious that the deviations based on the ℓ_1 norm provides better estimates in most of the tested cases. Namely, the procedure is repeated 1000 times (different sets of measurements are used) and the results have shown that in the presence of impulse noise, the ℓ_2 norm based *GD* fails to detect all signal components in 25% of cases (two components are detected correctly, while the third component is detected on the wrong frequency position), which is decreased to almost 6% in the case when the algorithm uses ℓ_1 norm based degradations. Of course, this is not the case when components are equal in amplitudes.

Example 5: The noniterative version of the proposed reconstruction procedure is applied to the signal in the form:

$$x(n) = 1.3e^{j64\pi n/N} + 1.5e^{j116\pi n/N} + 1.4e^{j256\pi n/N} + 1.2e^{j146\pi n/N}$$

In addition to four components, signal contains additive pulses of normal noise with a large standard deviation $\sigma = 10$ located at 25% random positions within the signal. The set of measurements contains 50% of randomly chosen signal samples out of $N = 256$. The threshold condition is $k_{0i} = \arg\{V < 0.98\mathrm{median}\{GD(k)\}\}$, while $GD(k)$ is considered in the form of errors based on the ℓ_1 and ℓ_2 norm. Again, in this case, values k_{0i}, $i = 1, 2, ..., K$ might have a slightly higher number of elements than the exact number of signal components, but it does not affect the precision of reconstruction, since in the resulting DFT vector **X**, the insignificant components will be zero valued. The reconstructed versions of signal and its DFT are shown in Fig. 8. The noise-free signal with 50% of randomly positioned available samples, analyzed by the standard DFT forms with the loss function $F(e) = |e|^2$, is shown in the first column in Fig. 8. The CS signal from the first column with a strong impulse noise at 25% of randomly positioned samples is shown in the second column (it is analyzed by the standard DFT forms with the loss function $F(e) = |e|^2$). The CS noisy signal from the second column, analyzed by the robust DFT forms with the loss function $F(e) = |e|$ is shown in the third column. The Fourier transforms of CS signals are presented in the first row; the generalized deviations are depicted in the second row, while the reconstructed transforms obtained using the proposed method are given in the last row. In the case of robust estimation, the set of available data is reduced, after frequencies k_{0i}, $i = 1, 2, ..., K$ are determined, to 8 samples around the median position. The same procedure may be repeated with an assumed number of

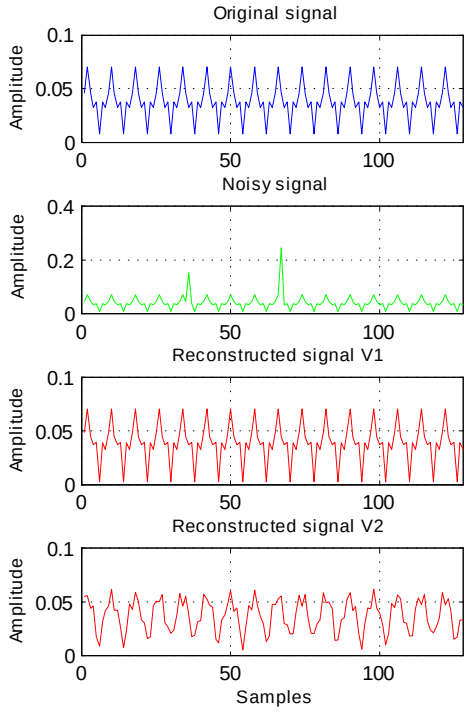

Fig. 6. Original and reconstructed versions of signal in time domain

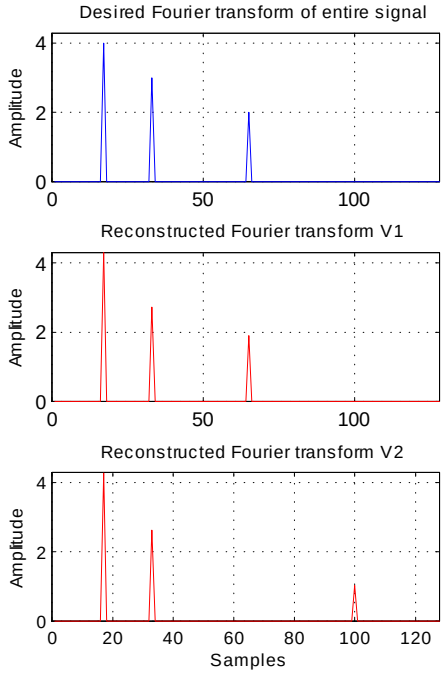

Fig. 7. Original and reconstructed versions of the Fourier transform

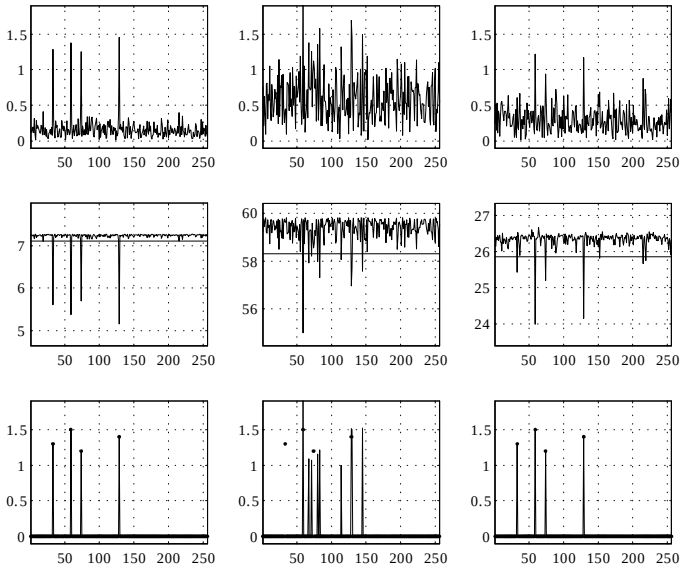

Fig. 8. The results of signal reconstruction

components K, without using a threshold.

VI. Conclusion

The reconstruction of signals from an incomplete set of its samples is considered. Starting from the formulation of robust representations which is based on the loss function applied to estimation errors, the algorithm discriminates between signal and non-signal components. Namely, it has been shown that the variances of estimation errors, calculated for each frequency separately, can be used as a reliable indicator whether there is a signal component at the observed frequency or not. The algorithm detects and then sets aside one by one component depending on the variance and the component amplitude. As shown on the examples, the algorithm provides efficient results with low mse values even when the components amplitudes differ. As a future work, the proposed approach can be extended to signals with non-stationary phases, by using the short-time Fourier transform and applying the same concept on a window-by-window basis. The same approach could be used in on the polynomial Fourier transforms to process the signals with polynomial phase function.

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