An algorithm for micro-Doppler period estimation

Miloš Daković, *Member*, *IEEE*, Miloš Brajović, *Student Member, IEEE,* Thayanathan Thayaparan, *Member, IEEE,* Ljubiša Stanković, *Fellow, IEEE*

***Abstract* — Radar micro-Doppler signatures are important in identification of properties of unknown targets. Many applications in real world scenarios require estimation of micro-Doppler parameters. Micro-Doppler oscillation period is one of its important parameters. A new algorithm for micro-Doppler oscillation period estimation based on time frequency signal analysis is proposed. Its functionality and theoretical considerations are evaluated using synthetic data, and numerical results are presented.**

***Keywords* — micro-Doppler effect, period estimation, radar signal processing, time-frequency signal analysis.**

I. INTRODUCTION

ANALYSIS of non-stationary signals can be performed using time-frequency representations and techniques [1]-[7]. Those signals have time-varying spectral contents. Signal parameters estimation is one of application fields of time-frequency signal analysis.

Time-frequency (TF) analysis provides a joint time-frequency representation (TFR) of a signal [1]- [4]. Time-frequency signal representations, such as Wigner distribution (WD) and spectrogram are concentrated around the signal's instantaneous frequency (IF) [1], [3]. Instantaneous frequency is defined as the first derivative of the signal's instantaneous phase [1], [2]. IF estimation is a challenging topic in the signal processing [2].

Important application field of TF signal analysis is in processing of radar signals. Radar micro-Doppler (m-D) signatures are used to identify properties of unknown targets [8]-[11]. Mechanical vibration or rotation of a target, or its parts induces frequency modulations to the regular Doppler shift on the returned signal, which can be detected as variations of the target's Doppler frequency, known as micro-Doppler [8], [11]. Since the micro-Doppler is a unique signature of a target, it can be used for determination of the target's properties [8]. TF analysis techniques can be used for extraction of the m-D characteristics [8], [9]. Details on the m-D effect may be found in [11]. Since the m-D must be separated from a target's rigid body before it can be analyzed, we refer reader to some m-D separation and extraction methods described in [8] and [10]. Since it is caused by rotation (or vibration) of the target or its parts, period (frequency) of m-D oscillations is one of its most important parameters.

In this paper, an algorithm for the m-D frequency (period) estimation is proposed. Algorithm is based on TF signal analysis concepts. Theoretical consideration are covered with numerical results with synthetically generated signals.

The paper is organized as follows. A brief description of basic concepts of time-frequency signal analysis and micro-Doppler effect in radar signals, along with basic ideas of the proposed algorithm is given in Section 2. Section 3. describes the proposed algorithm. Numerical results are given in Section 4. while the concluding remarks are given in the Section 5.

II. TIME FREQUENCY SIGNAL ANALYSIS AND MICRO-DOPPLER PERIOD ESTIMATION

Consider a continuous frequency modulated (FM) signal $s(t)$ defined by:

$$s(t) = A(t)e^{j\phi(t)}. \tag{1}$$

We assume that the amplitude variations are small. Instantaneous frequency (IF) can be defined as the first derivative of the signal's instantaneous phase:

$$\omega(t) = \frac{d\phi(t)}{dt}. \tag{2}$$

Mechanical vibration or rotation of a target induce frequency modulation on the signal returned to a radar. It causes generation of sidebands around the Doppler frequency shift of the target's body. In order to illustrate our considerations, we will introduce a simplified model of a returned radar signal with the m-D effect [8] :

$$s(t) = \exp(j2\pi f_0 t + j\frac{A}{f_v}\sin(2\pi f_v t + \theta)). \tag{3}$$

Thus, the IF of the returned signal can be written as:

$$f(t) = f_0 + A\cos(2\pi f_v t + \theta) = f_0 + f_p(t) \tag{4}$$

where f_0 is the Doppler frequency caused by the target motion and the second part, $f_p(t) = A\cos(2\pi f_v t + \theta)$ is the Doppler frequency caused by vibrating (or rotating) parts of the target. Our goal is to estimate the target rotation or vibration rate f_v. As it will be shown, our algorithm might be used not only for estimation of a sinusoidal, but for estimation of any periodic part of the IF

M. Daković, Elektrotehnički fakultet, Džordža Vašingtona bb, 81000 Podgorica, Montegro (e-mail: milos@ac.me)

M. Brajović, (contact author), Elektrotehnički fakultet, Džordža Vašingtona bb, 81000 Podgorica, Montenegro (phone: 382-69-486639, e-mail: milosb@ac.me)

T. Thayaparan, Radar Applications and Space Technology - Defense R&D Canada, Ottawa 3701 Carling Avenue, Ottawa, Ontario, Canada K1A 0Z4 (e-mail: Thayananthan.Thayaparan@drdc-rddc.gc.ca)

Lj. Stanković, Elektrotehnički fakultet, Džordža Vašingtona bb, 81000 Podgorica, Montenegro (e-mail: l.stankovic@ieee.org)

$f_p(t)$ in the eq. (4), which corresponds to the m-D.

The instantaneous frequency of the radar's returned signal can be represented using TF signal analysis as it is described in [1]-[6]. It is known that the signal's energy is concentrated around the instantaneous frequency in the time-frequency plane [1]-[6].

Let us consider a discrete-time noisy signal $x(n) = s(n) + \nu(n)$, where the deterministic signal $s(n)$ is the corresponding discrete version of continuous frequency modulated (FM) signal $s(t)$ given in (1). Also, assume that the noise $\nu(n)$ is a white, Gaussian, complex, stationary, zero-mean process, with independent real and imaginary parts having equal variances σ^2 (variance of noise is $2\sigma^2$).

Using TF analysis, a TF representation $TFR(n,k)$ of the signal $x(n)$ is calculated.

Any discrete time-frequency representation is a two-dimensional function of time and frequency, i.e. a $N \times L$ matrix, where N is the length of the signal, and L is the number of discrete frequency points. The micro-Doppler is included as the periodic part of the IF in $TFR(n,k)$. The basic idea behind our algorithm lies in fact that, since the m-D is a periodic function, its parts are periodically repeated over time, and that repetition might be detected directly from the TF plane. It can be done if we use a part of the time frequency plane, slide it across time index *n,* and compare with every equivalent part of the $TFR(n,k)$, detecting the time points of its values repetition in the $TFR(n,k)$.

Let us give a more detailed description of the basic ideas of the proposed algorithm. First, consider the case of FM signal $x(n) = s(n)$ without the noise. A part of the considered time-frequency representation $TFR(n,k)$, which is wide enough in time, is chosen. Then it is slided in time n across the original $TFR(n,k)$. That part is multiplied with the corresponding part of the original time-frequency representation of the same size for the given instant *n*, and the sum of all points of the obtained product is calculated. Described process is repeated for every instant *n*. Mentioned summation will be used as a similarity measure between the selected part and the corresponding part of the original $TFR(n,k)$ for current instant n, because the sum will be largest when the values in these parts are the same (or almost the same). The reason for previous conclusion lies in fact that time-frequency representations are concentrated around the IF, which means that their values are largest around it, while all other points are close to zero. If the IF in the selected (sliding) part completely overlaps the IF in the part of $TFR(n,k)$ matrix for current instant n, then every point around the IF will be multiplied with the same point value. If they do not overlap, most points in one matrix will be multiplied with small or zero values from the other matrix. We used these facts to derive the proposed algorithm.

Now consider previously defined noisy FM signal $x(n) = s(n) + \nu(n)$. It is well known [1], [12] that the white Gaussian noise will be uniformly distributed over the whole time frequency plane. Since all points, including those around the IF will be uniformly corrupted with noise, it does not change the fact the similarity measure will be still largest when the instantaneous frequencies of considered TFR's parts overlap completely. As it will be shown later, it means that even in the presence of a strong noise, algorithm relatively precisely estimates the m-D period T_v.

III. ALGORITHM

The proposed algorithm might be described in next few steps:

Step 1. Calculate the time-frequency representation $TFR(n,k)$ of signal $x(n)$. $TFR(n,k)$ is a $N \times L$ matrix.

Step 2. Select a part of $TFR(n,k)$, which is in fact a $p \times L$ matrix, consisted of first p columns of the matrix $TFR(n,k)$. For every instant n, calculate the similarity measure:

$$M(n) = \sum_{l=0}^{p-1} \sum_{k=0}^{L-1} TFR(l,k) TFR(n+l,k) \qquad (5)$$

Step 3. Find maxima Max in $M(n)$ using the threshold Θ with condition given by:

$$Max > \Theta \cdot \sum_{l=0}^{p-1} \sum_{k=0}^{L-1} TFR(l,k) \qquad (6)$$

Step 4. Refinement: for each maximum, interpolate the points around maximum with a parabola, and find interpolated maximum position.

Step 5. The distance between two successive maxima gives the m-D discrete period T_v. To calculate this period in seconds [s], it should be multiplied with the sampling period Δt, which was used in continuous signal $s(t)$ discretization.

Number of columns p in matrix A, should be chosen such that for a signal of approximately m periods, it is approximately $25/m$ percent of the number of columns N of matrix $TFR(n,k)$. That value is obtained heuristically. It is easy to obtain the approximate number of periods m by directly observing the considered time-frequency representation. For example, if the number of periods of IF in the considered TFR (for signal of length *N*) is about 2, the parameter p should be approximately 10% of the number of columns in matrix $TFR(n,k)$, while in case of about 4 periods, it should be approximately 5%, etc.

Threshold Θ selection depends on signal-to-noise ratio (SNR) of the noisy signal $x(n)$. Larger values of threshold can be used for low SNRs, while it should be decreased with the increase of SNR.

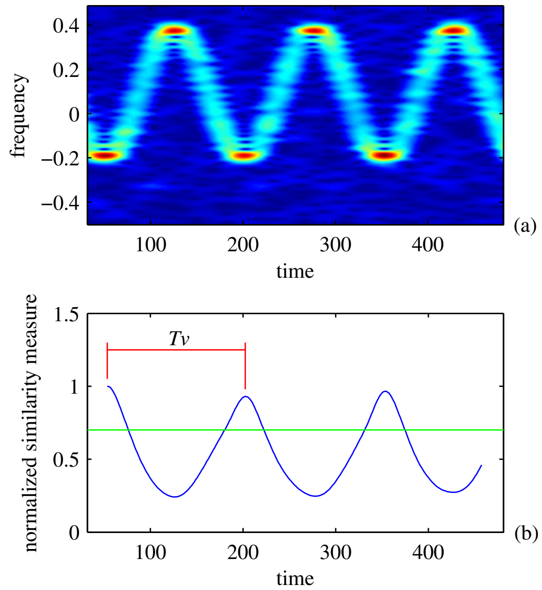

Fig. 1. Micro-Doppler period estimation in white noise environment: a) |*STFT*(*n*, *k*)| of the signal from the Example 1; b) Obtained normalized similarity measure

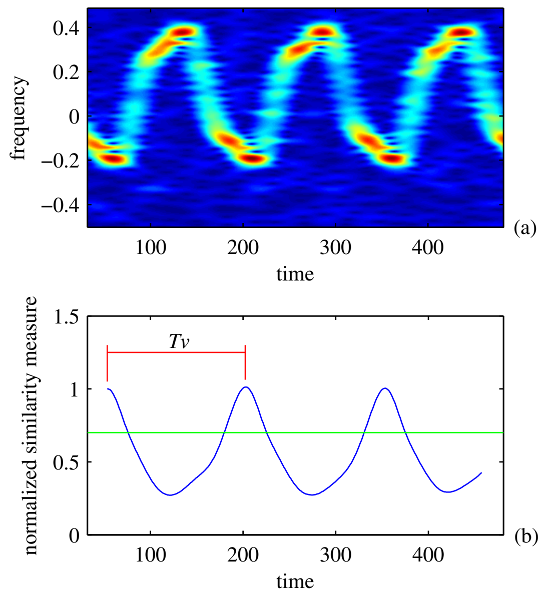

Fig 2. Micro-Doppler period estimation in white noise environment: a) |*STFT*(*n*, *k*)| of the signal from the Example 2. b) Obtained normalized similarity measure

IV. EXAMPLES

In our examples, we consider the absolute value of discrete Short-Time Fourier Transform as an example of time-frequency representation, due to its practical and theoretical importance.

For signal $x(n)$, STFT is defined by [1]-[4]:

$$STFT(n,k)=\sum_{m=-N/2}^{N/2-1} x(n+m)w(m)e^{-j\frac{2\pi}{N}km}$$

where the window function w(n) has the width N. Synthetic signals with characteristics of real radar signals with m-D effects are used in our examples.

Example 1. Consider a noisy sinusoidal FM signal with periodic instantaneous frequency part (which represents micro-Doppler):

$$x(n)=\exp(j0.6\sin(2\pi f_v n+\pi/3)/f_v/2+j0.2\pi n)+\nu(n)$$

of length $N=512$ and with discrete modulation frequency $f_v=1/150$ (m-D frequency). Signal is corrupted with white complex Gaussian noise, and SNR is 10 dB. Observing the |*STFT*(*n*, *k*)| of signal $x(n)$ shown on Fig 1a, it may be concluded that the approximate number of m-D periods is 3. Notice that from this TF representation of noisy signal actual micro-Doppler period could not be precisely determined, due to noise caused corruption and window caused low concentration. Estimated number of columns is $p=43$. Normalized similarity measure as a function of time is shown on Fig 1b. The distance between first two successive maxima is $T_v=149.3451$.

Example 2. Nosy FM signal $x(n)$ with m-D given as a:

$$x(n)=\exp(j0.6\sin(2\pi n/f_v+\pi/3))\times$$
$$\times\exp(j0.1\sin(6\pi n/f_v+\pi/3)/(f_v/2)+j0.2\pi n)+\nu(n)$$

is considered, with $f_v=1/150$. SNR is still 10, and noise is white, complex and Gaussian. Fig 2a shows |*STFT*(*n*, *k*)| of considered noisy signal $x(n)$. In this example it is also obvious that the determination of IF period is not simple by visually observing given TF representation. Normalized similarity measure as a function of time is shown on Fig 2b, and the required period $T_v=149.3450$ is the distance between two successive maxima, as labeled on the figure.

Example 3. Given the |*STFT*(*n*, *k*)| of a noisy FM signal with periodic IF on Fig 3a, proposed algorithm was used for m-D period T_v estimation. Complex, white, Gaussian noise corrupted the FM signal, and the SNR is 35 dB. Unlike in previous two examples, besides the maximum values of normalized similarity measure as a function on time, significant values in other time positions appear, as shown on Fig 3b. This example illustrates the importance of the suitable threshold selection. The selected threshold prevented the influence of mentioned significant values in discrete period T_v estimation. Estimated value is $T_v=149.9558$.

Example 4. Multicomponent [1]-[4] noisy FM signal $x(n)=x_1(n)+x_2(n)+x_3(n)+\nu(n)$ is considered, with $f_v=1/150$. Signal is consisted of one linear frequency modulated signal whose IF is non-periodic:

$$x_1(n)=0.5\exp(-j\pi 0.6n^2/512+j0.5\pi n),$$

one complex sinusoid with frequency 0 (constant IF): $x_2(n)=0.5\exp(j0)$, and one component with periodic IF:

$$x_3(n)=\exp(j0.6\sin(2\pi f_v n+\pi/3))/(f_v/2)\times$$
$$\times\exp(j0.1\sin(6\pi f_v n+\pi/3)/3)/(f_v/2)\times\exp(j0.2\pi n).$$

Noise is white, complex, Gaussian, and SNR=10 dB. |*STFT*(*n*, *k*)| of noisy FM signal $x(n)$ is presented on Fig 4a, while the normalized similarity measure is shown on Fig 4b. As it is expected, non-periodic components of signal $x(n)$ distorted maxima of similarity measure, but, still with a suitable threshold selection, the algorithm is able to estimate the period. Estimated micro-Doppler period is $T_v=150.0735$.

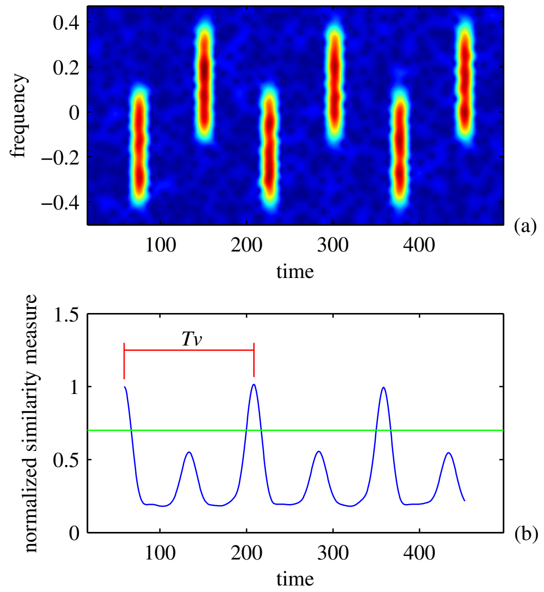

Fig 3. Micro-Doppler period estimation in white noise environment: a) |*STFT*(*n*, *k*)| of signal from the Example 3; b) Obtained normalized similarity measure

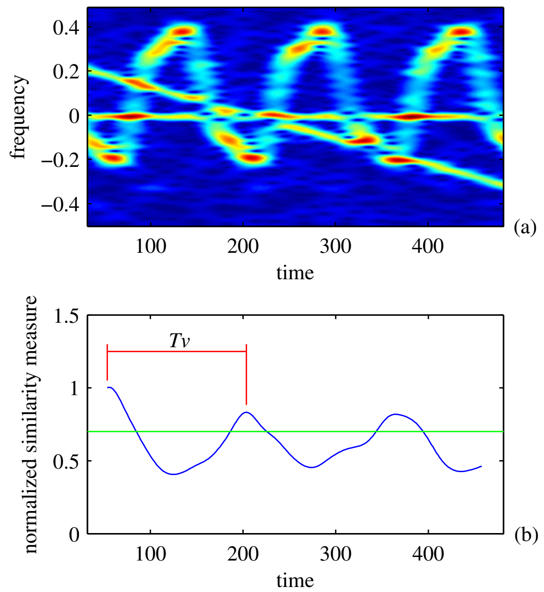

Fig 4. Micro-Doppler period estimation in white noise environment: a) |*STFT*(*n*, *k*)| of signal described in Example 4; b) Normalized similarity measure

Example 5. Consider the discrete time-limited chirp signal $x(n)$ with $f_v = 1/150$:

$$s(n) = \begin{cases} \exp(j0.6\pi f_v n^2 - j0.5\pi n), 0 \le n \le T_v \\ 0, \text{ elsewhere} \end{cases}$$

Discrete signal $x(n)$, defined as a periodic noisy array consisted of 5 signals $s(n)$:

$$x(n) = \sum_{m=0}^{4} s(n - m / f_v) + \nu(n)$$

is given. Noise is white, complex, Gaussian, with SNR=10dB. Fig 5a shows |*STFT*(*n*, *k*)| of signal $x(n)$ in first 512 points, while the Fig 5b shows normalized similarity measure. The m-D period $T_v = 149.4715$ is estimated as a distance between two successive maxima.

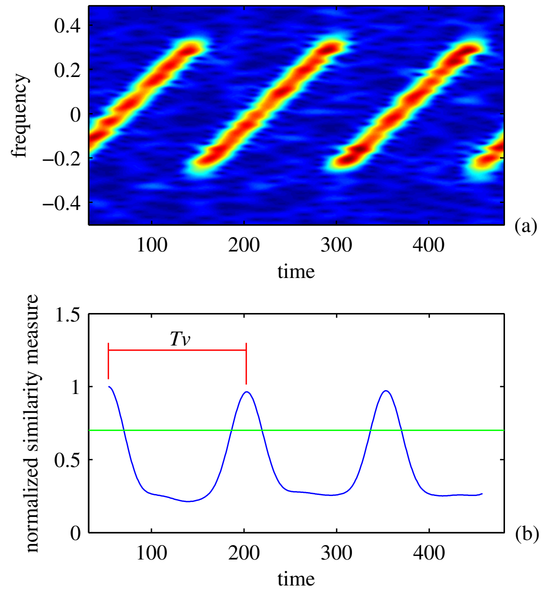

Fig 5. Micro-Doppler period estimation in white noise environment: a) |*STFT*(*n*, *k*)| of signal described in Example 5; b) Normalized similarity measure

V. Conclusion

An algorithm for micro-Doppler period (frequency) estimation is proposed. Micro-Doppler parameters estimation is important in radar signal processing since it can be used for identifying properties of unknown targets. Numerical results illustrate the algorithm's simplicity and effectiveness, showing its practical potentials in radars.

References

[1] L. Cohen, "Time-Frequency distributions - a review", *Proc. IEEE,* vol 77, no.7, pp. 941-981, July 1989.

[2] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal –Part 1: Fundamentals", *Proc. of the IEEE,* vol. 80, no.4, pp.519-538, April 1992.

[3] F. Hlawatsch and G.F. Boudreaux-Bartels, "Linear and quadratic time-frequency signal representations", IEEE Signal processing Magazine, pp. 21-67, April 1992.

[4] L. Stanković, "A method for time-frequency analysis", *IEEE Trans. on Signal Processing,* Vol. 42, pp. 225-229, 1994.

[5] V. Katkovnik, LJ. Stanković, "Instantaneous frequency estimation using the Wigner distribution with varying and data-driven window length", *IEEE Trans. on Signal Proc,* vol.46, no.9,pp.2315-2326, Sept.1998.

[6] LJ. Stanković, V. Ivanović, Z. Petrović, "Unified approach to the noise analysis in the spectrogram and the Wigner distribution", *Ann.des telecom.,* vol.51, No.11-12, pp. 585-594, Nov./Dec. 1996.

[7] T. Thayaparan, LJ. Stankovic, M. Dakovic, V. Popovic, "Micro-Doppler parameter estimation from a fraction of the period," *IET Signal Processing*, Vol. 4, No. 1, 201 - 212, Jan. 2010.

[8] T. Thayaparan, LJ. Stankovic, I. Djurovic, "Micro-Doppler-based target detection and feature extraction in indoor and outdoor environments", *Journal of The Franklin Institute*, Vol. 345, No. 6, pp. 700-722, Sept. 2008.

[9] T. Thayaparan, S. Abrol, E. Risebrough, LJ. Stankovic, D. Lamothe, G. Duff, "Analysis of Radar Micro-Doppler Signatures From Experimental Helicopter and Human Data," *IEE Proc. Radar, Sonar and Navig.,*, Volume 1, Issue 4,, pp. 289-299, Aug. 2007.

[10] LJ. Stankovic, T. Thayaparan, I. Djurovic, "Separation of Target Rigid Body and Micro-Doppler Effects in ISAR Imaging," *IEEE Trans. on Aerospace and El. Systems*, Vol 42, No. 7, pp. 1496-1506, Oct. 2006.

[11] V. C. Chen, F. Li, S.-S. Ho, and H. Wechsler, "Micro-Doppler effect in radar: phenomenon, model , and simulation study", Vol. 42, No. 1, pp. 2-21, *IEEE Trans. on Aero. and El. Systems,* 2006.

[12] A.Papoulis, Signal analysis, McGraw Hill, New York, 1977.