Compressive sensing reconstruction of signals with sinusoidal phase modulation: application to radar micro-Doppler

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***Abstract* — The algorithm for compressive sensing reconstruction of signals with sinusoidal phase modulation is proposed. The signal is firstly demodulated and reduced to a set of sparse sinusoids, while the reconstruction is achieved by exploiting the sparsity in the DFT domain. The demodulation process is based on the parameters search method which also employs specific compressive sensing reconstruction procedure to detect the exact parameters sets. The procedure is tested on the multicomponent signals with sinusoidal modulation that often appear in radar communication as a micro-Doppler part reflected from fast rotating scatterers.**

***Keywords* —signal reconstruction, compressive sensing, random undersampling, sinusoidal phase modulation**

I. INTRODUCTION

NOWADAYS the Compressive sensing concept (CS) has been widely accepted as a new alternative approach to signal acquisition [1]-[7]. This concept brings the possibility to deal with highly undersampled data, which is especially important in the situations when sensing, storing and transmission of large data amount is difficult, expensive and not feasible. The conventional sampling theorem states that the signal has to be sampled at the rate at least twice higher than the maximal frequency in order to preserve the signal information. In the CS scenarios, the signal can be exactly recovered from a small number of randomly chosen measurements, even when the number of measurements M is far below the number of samples required by the sampling theorem. The CS exploits two important properties: 1) the signal sparsity when represented in appropriate basis; 2) incoherence between the measurement matrix and sparsity basis [1]. As a commonly used example of sparse signal model, we can observe the sum of complex sinusoids which can be exactly reconstructed using the orthonormal Fourier basis [8],[9]. The signals encountered in real applications may have quite different nature. For instance, although the rigid body components in radar applications could be sinusoids-like, the micro-Doppler components that originate from fast rotating target parts can be modeled by the sinusoidal phase modulation [10],[11]. Now, having in mind that such signals are not generally sparse in the commonly known frequency domains, the CS application and reconstruction becomes a challenging task.

In this paper we propose a simple method for the reconstruction of signal characterized by sinusoidal phase modulation. The method is based on components demodulation technique which is further combined with the threshold based sparse components detection proposed in [8]. The demodulation process aims at detecting signal phase parameters. Ideally, in the case of monocomponent signal, the demodulation will result in a single sinusoid, i.e., single spike in the Fourier transform domain. However, when dealing with multicomponent signals, the demodulation term corresponding to one component will interfere with other components. In this case, an efficient solution for components detection is achieved using the threshold based procedure. Once the signal parameters are determined as a result of demodulation, the problem is reduced to the reconstruction of sparse sinusoids.

The paper is organized as follows. The theoretical background about the Compressive sensing theory is given in Section II. The problem formulation and the proposed solution for the CS-based reconstruction of signal with sinusoidal phase modulation are given in Section III. The experimental results are given in Section IV, while the concluding remarks are given in Section V.

II. THEORETICAL BACKGROUND - COMPRESSIVE SENSING

The CS concept has been used as an alternative way to sample/acquire signals in various signal processing applications. This concept is based on the premise that the signal can be completely recovered from a small and randomly chosen set of measurements when two important requirements are met. The first requirement is related to the signal sparsity, meaning that the observed signal can be represented by a small number of non-zero coefficients in certain transform domain. For instance, a signal $s(t)$ can be represented in basis $\mathbf{\Psi}$ using basis vectors ψ_i:

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$$s(t)=\sum\nolimits_{i=1}^{M} S_i(\omega)\psi_i(\omega), \qquad (1)$$
$$\mathbf{s}=\mathbf{\Psi S}.$$

If the vast majority of transform domain coefficients within the vector **S** are zero (or close to zero), we say that **S** is sparse representation of **s**. The commonly used transform domains are Discrete Fourier Transform - DFT, Discrete Wavelet transform - DWT, or time-frequency domain [3],[4],[12]-[14].

The second requirement imposes incoherent measurement process. Signal measurements are usually acquired in the domain where signal have "dense" representation [2]:

$$\mathbf{y}=\mathbf{\Phi s}\,, \qquad (2)$$

where $\mathbf{\Phi}$ is a measurement matrix ($N\times M$) that models random measuring process, while **y** is the resulting measurement vector. Based on (1) and (2) we can write the set of equations in the matrix form as follows:

$$\mathbf{y}=\mathbf{\Phi s}=\mathbf{\Phi\Psi S}=\mathbf{\Theta S}\,, \qquad (3)$$

where the measurement matrix $\mathbf{\Phi}$ must be incoherent with the basis matrix $\mathbf{\Psi}$. Therefore, a K sparse signal can be completely characterized by using just $N>K$ measurements, where $N<<M$ holds and M is the number of samples defined by the sampling theorem. Based on this small set of randomly chosen samples, the entire signal can be reconstructed using powerful mathematical algorithms [1],[4]-[3]. The system of equations (3) is undetermined since it consists of N equations with M unknowns ($N<<M$). Hence, in order to achieve an optimal solution, the optimization algorithms are employed to search for the sparsest solution using the ℓ_1 – minimization[1] [15]-[17]:

$$\hat{\mathbf{S}}=\min\|\mathbf{S}\|_{\ell_1} \quad s.t. \;\; \mathbf{y}=\mathbf{\Theta S}\,. \qquad (4)$$

III. Reconstruction of CS signals with sinusoidal phase modulation

A. Signal sparsity using sinusoidal phase demodulation

One of the common examples of sparse signals is a pure sinusoidal component that is analyzed in the DFT domain. However, when dealing with signals with nonlinear phase functions such as the considered sinusoidal phase modulation, the main challenge is to determine the domain of signal sparsity. Let us observe the signal s, consisting of a sum of K components with sinusoidal phase modulation:

$$s(m)=\sum_{i=1}^{K}s_i(m)=\sum_{i=1}^{K}A_i e^{j\frac{2\pi}{M}b_i\sin(2\pi a_i m/M)+j\frac{2\pi}{M}c_i m}. \qquad (5)$$

The coefficients a, b and c are assumed to be bounded integers, m is a discrete time parameter, while M is the total number of signal samples (signal length). The components amplitudes are denoted by A_i. The DFT of signal s, given by:

$$S(k)=\sum_{m=0}^{M-1}\sum_{i=1}^{K}A_i e^{j\frac{2\pi}{M}b_i\sin(2\pi a_i m/M)+j\frac{2\pi}{M}mc_i}e^{-j\frac{2\pi}{M}mk}, \qquad (6)$$

is not sparse in DFT (neither in other commonly used sparsity domains DCT or DWT, etc.) and as such is not amenable to CS applications. In that sense, we propose a simple technique to demodulate the signal of interest and reduce the problem to the reconstruction of sinusoids in the DFT domain. The term:

$$v(m)=e^{-j\frac{2\pi}{M}b\sin(2\pi am/M)} \qquad (7)$$

is introduced to compensate the nonlinear phase part of signal components. Consequently, when the set of observed parameters (a,b) within the demodulation term is chosen to match with at least one of the phase parameters $(a_1,b_1),(a_2,\;b_2),\ldots,(a_K,b_K)$, the corresponding signal component will be demodulated and reduced to sinusoid, i.e., for the i-th component we will have:

$$d_i(m)=A_i e^{j\frac{2\pi}{M}mc_i}\,. \qquad (8)$$

It is important to note that the demodulation procedure needs to be applied using the component-by-component strategy. Namely, in order to demodulate the i-th component, the parameter search procedure is performed:

for $a\in(a_{min},\,a_{max})$ *and* $b\in(b_{min},\,b_{max})$
if $DFT\{s(m)\cdot v(m),\; m=0,\ldots,N-1\}$ *has a spike at* $k=c_i$
then $a=a_i$, $b=b_i$

In this case, the spectrum $S(k)$ is highly concentrated at $k=c_i$. Otherwise for $a\neq a_i$ or $b\neq b_i$ the spectrum is dispersed. Here, the assumption is: $M\left|A_{\min}\right|>\sum_{i=1}^{K}\left|A_i\right|$.

B. Proposed signal reconstruction procedure

In order to define the CS problem of interest, the demodulated version of signal **s** is defined in the vector form as follows:

$$\mathbf{x}=\mathbf{s}\circ\mathbf{v}\,, \qquad (9)$$

such that **x** contains samples of **s** multiplied by demodulation terms (the component-wise product is denoted by the operator ($\circ$)):

$$v(m)=\exp -j\frac{2\pi}{M}b_i\sin\left(2\pi a_i m/M\right),\, m\in(0,..,M-1)\,.$$

Then the DFT of demodulated signal can be written as:

[1] The sparsest solution of the minimization problem is originally defined using ℓ_0-norm, but the in the practical applications, the optimization problems becomes NP-hard.

$$\mathbf{x}=\mathbf{\Psi}\mathbf{X}\,, \qquad (10)$$

where $\mathbf{X}$ is the vector of DFT coefficients, while $\mathbf{\Psi}$ is $M\times M$ DFT matrix. For a chosen pair of parameters (a,b) in $\mathbf{v}$ that is equal to the (a_i,b_i) in $\mathbf{s}$, $\mathbf{X}=\mathbf{X_i}$ is characterized by the i-th sinusoidal component at the frequency c_i that becomes dominant in the spectrum. Now, assume that $\mathbf{x}$ is compressive sampled signal, represented by an incomplete set of N randomly chosen samples. Thus, instead of $\mathbf{s}$ we are dealing with a measurement vector $\mathbf{y}$ obtained using the incoherent measurement matrix $\mathbf{\Phi}$ ($N\times M$):

$$\mathbf{y}=\mathbf{\Phi}\mathbf{x}=\mathbf{\Phi}(\mathbf{s}\circ\mathbf{v})\,. \qquad (11)$$

Now, using (10) the CS problem can be formulated as:

$$\mathbf{y}=\mathbf{\Phi}\mathbf{\Psi}\mathbf{X}=\mathbf{\Theta}\mathbf{X}\,, \qquad (12)$$

where $\mathbf{\Theta}=\mathbf{\Phi}\mathbf{\Psi}$. In the case when the searching parameter $a=a_i$ and $b=b_i$ then the DFT vector $\mathbf{X}$ can be approximately observed as a demodulated version of the i-th signal component $\mathbf{X}=\mathbf{X_i}$ that can be recovered by solving the l_1 -norm minimization problem in the form:

$$min\left\|\mathbf{X}_i\right\|_{\ell_1} \;\; s.t.\;\; \mathbf{y}=\mathbf{\Theta}\mathbf{X}_i\,.$$

Nevertheless, since we consider multicomponent signals, in real situations it is necessary to deal also with the terms resulted from cross-component multiplication. This product terms will be noise-like and spread in the spectrum. Consequently, we need to provide the reconstruction of the i-th signal component, neglecting the influence of other spread terms. For that purpose we use the threshold based single iteration algorithm proposed in [8]. The threshold detects only the sparse signal components which allows us to reconstruct just the dominant i-th component $\mathbf{X}=\mathbf{X_i}$.

In the case $(a,b)\neq(a_i,b_i)$ for $\forall i$ and $N<<M$ there is no any dominant component and zero value is obtained as the output of reconstruction.

The exact values of the components amplitudes (vector $\mathbf{A}$) are obtained as a solution of the following equations (in matrix form):

$$\mathbf{A}=(\Delta^{\mathbf{H}}\times\Delta)^{-\mathbf{1}}\times\Delta^{\mathbf{H}}\times\mathbf{y} \qquad (13),$$

where,

$$\Delta=\begin{bmatrix} e^{j\frac{2\pi}{M}b_1(\sin(2\pi m_1 a_1)+m_1c_1)} & \dots & e^{j\frac{2\pi}{M}b_K(\sin(2\pi m_1 a_K)+m_1c_K)} \\ e^{j\frac{2\pi}{M}b_1(\sin(2\pi m_2 a_1)+m_2c_1)} & \dots & e^{j\frac{2\pi}{M}b_K(\sin(2\pi m_2 a_K)+m_2c_K)} \\ \dots & & \dots \\ e^{j\frac{2\pi}{M}b_1(\sin(2\pi m_N a_1)+m_Nc_1)} & \dots & e^{j\frac{2\pi}{M}b_K(\sin(2\pi m_N A_K)+m_Nc_K)} \end{bmatrix}$$

The rows of $\mathbf{\Delta}$ correspond to measurements instants $(m_1,m_2,\ldots,m_N)$, while columns correspond to frequencies $\boldsymbol{a_i}$, for $i=1,\ldots,K$.

IV. Experimental Evaluation

Example 1: Consider the multicomponent signal in the form:

$$x(t)=4e^{-j3sin\left(\frac{0.35T2\pi t}{2}\right)+j2\pi 32t}+2e^{-j5sin\left(\frac{0.05T2\pi t}{2}\right)+j2\pi 160t}$$

where amplitudes A_1=4 and A_2=2 , t=[-1/2,1/2] with step Δt=1/1024 and T=32. The total signal length is M=1024 samples. Also, the signal is represented using K=24 randomly chosen measurements (2,35 % of the total signal length). This type of signals usually appears in radar applications, as a micro-Doppler component that originates from the fast rotating target parts. Note that the signal can be generally written in the form:

$$x(t)=A_1e^{-jb_1sin\left(\frac{a_1T2\pi t}{2}\right)+j2\pi tc_1}+A_2e^{-jb_2sin\left(\frac{a_2T2\pi t}{2}\right)+j2\pi tc_2}.$$

In our case a_1=0.35, b_1=3, c_1=T=32 (for the first component), a_2=0.05, b_2=5, c_2=160 (for the second component). Therefore, we need to detect two different values of parameter a (a_1=0.35 and a_2=0.05) and two different values of signal parameter b (b_1=3 and b_2=5). The incomplete set of available samples is firstly multiplied by the corresponding exponential terms $e^{jb\sin(2\pi aTt/2)}$, in order to perform demodulation. Parameters a and b are changed iteratively within the sets: $a\in[a_{\min},a_{\max}]$, $b\in[b_{\min},b_{\max}]$. In each iteration (for different a and b), the signal reconstruction procedure based on the threshold [8] is performed. When the spectrum is spread ((a,b) do not match neither (a_1,b_1) nor (a_2,b_2)) the result of reconstruction is zero, as shown in Fig.1.

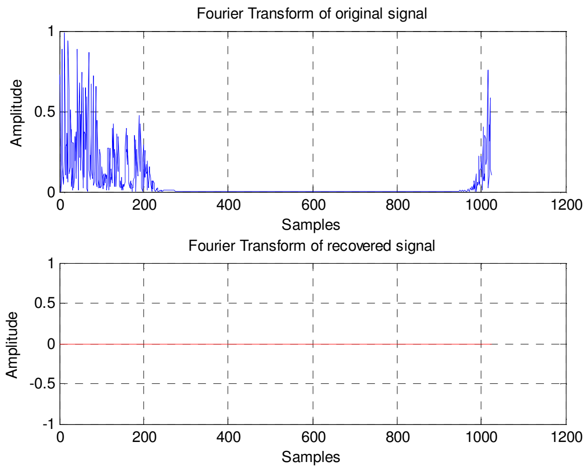

Fig. 1. DFT of demodulated signal and DFT of recovered signal when signal parameters are not matched for any (a_i,b_i)

When one pair of signal parameters is matched, the reconstruction procedure will provide sparse sinusoidal component at frequency c (c_1 or c_2 depending whether we detect (a_1,b_1) or (a_2,b_2)). The result for one matched pair (a_2=0.05, b_2=5) corresponding to the second signal component is shown in Fig.2 (similar result are obtained for the first component). It means that c can be determined as argument of the peak value in demodulated spectrum.

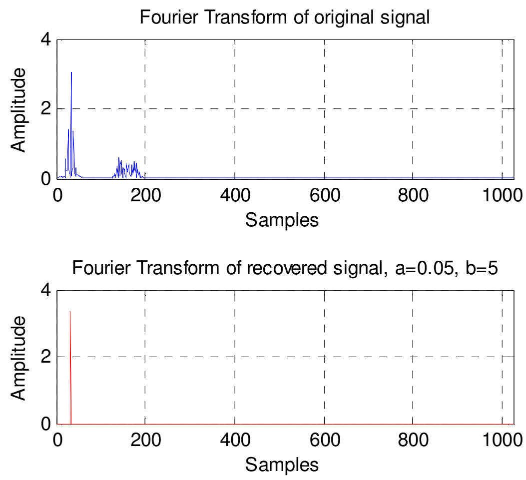

Fig. 2. DFT of demodulated signal and reconstructed DFT when signal parameters are matched for (a_2,b_2)

The reconstruction results (applied to demodulated signal) for several different pairs (a,b) are shown in Fig.3. The positions of two maxima correspond to (a_1=0.35, b_1=3), (a_2=0.05, b_2=5), i.e., the 16th and 25th trial. The reconstructed values for other pairs of (a,b) are zeros.

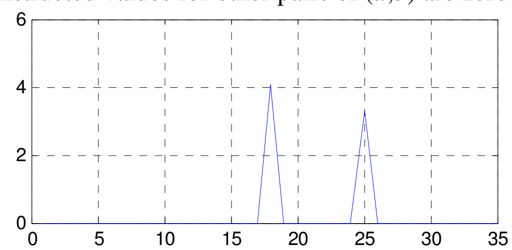

Fig 3. The reconstructed values of demodulate signal DFT obtained for different pairs (a,b)

After detecting the signal parameters a, b and c original amplitudes of demodulated components are recovered (A_1=4, A_2=2) and then the reconstructed signal components are obtained using re-modulation process, Fig. 4. The reconstructed components are identical to the original ones.

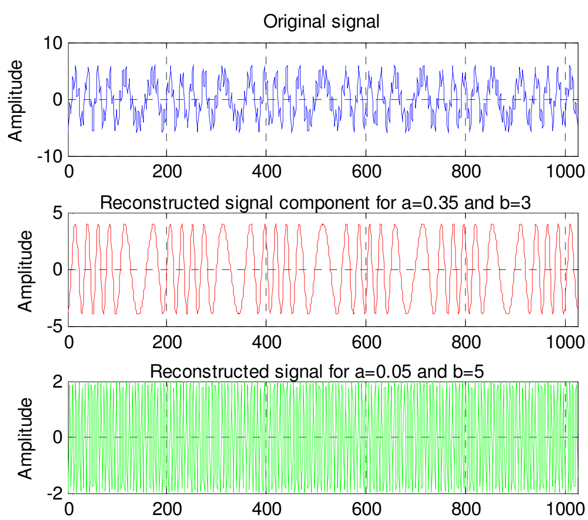

Fig 4. The reconstructed values of demodulate signal DFT obtained for different pairs (a,b)

V. Conclusion

The algorithm for CS reconstruction of signals with sinusoidal phase function is proposed. The considered signals are stationarized and sparsified during the demodulation process, which further allows reconstruction from small set of random measurements. The experiments show that the proposed method can be efficiently applied even in the case of highly undersampled signal (less than 3% of original signal length).

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