Instantaneous Frequency Estimation Using Ant Colony Optimization and Wigner Distribution

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Abstract - **Instantaneous frequency estimation of signals in a high noise environment is analyzed in the paper. An algorithm based on Ant colony optimization and Wigner distribution is proposed for solving the considered estimation problem. The proposed approach has been applied and tested on mono-component frequency-modulated signals. Numerical examples are given in order to demonstrate the algorithm's performances in the analyzed framework.**

Keywords - ant colony optimization; instantaneous frequency estimation; time-frequency signal analysis; Wigner distribution; digital signal processing

I. Introduction

During the last few decades, time-frequency signal analysis has drawn significant research attention in the area of signal processing [1]-[11]. A large number of time-frequency representations (TFRs) are developed for various applications of this scientific area. Estimation of the instantaneous frequency (IF) is an important application field of time-frequency signal analysis [1], [4]-[11]. Many TFRs have the property of concentrating signals' energy at and around the IF. This is the reason why the IF estimation problem formulation reduces to the determination of the TFR points with maximal values, in classical estimation approaches [1], [4], [8]. Wigner distribution (WD) has been widely used as an IF estimator of frequency modulated signals, since it highly concentrates signal in the time-frequency plane [6]-[10]. Accordingly, our study is restricted only to this representation as a starting point of the considered problem. The analysis of WD as an IF estimator was presented in detail in [6], [7], [10] and [11], where the estimation error sources were classified into four categories: bias, errors due to variations within the signal's auto-terms, frequency discretization based errors and errors caused by a high noise. The influence of a high noise has attracted a significant research attention, since the estimation error which it induces dominates over other error sources when it occurs, as explained in [7], [8] and [9]. The high noise causes the estimation error since its high values, outside of the auto-term, are for some time instants detected as maximum of the WD. Thus, this error is of impulse nature. The IF estimation in a high-noise environment is the main framework analyzed in this study.

The artificial ant colonies concept is a representative example of multi-agent tools for problem solving without centralized control. Since they are introduced in the area of artificial intelligence, the whole set of optimization techniques based on ant colony systems, widely known as Ant colony optimization, have been developed and applied in different scientific areas, especially where the hard-solving local optimization problems arise [12]-[18]. The artificial ant colonies are one of many concepts in the so called swarm intelligence, where a population of artificial agents forms a collective intelligence over a specific environment [12]. Important application fields include edge detection, pattern recognition and segmentation of digital images [14]-[17]. Ant colony optimization usage in the area of data-mining is described in [18]. This paper presents a modification of the Ant colony optimization algorithm used for edge detection of digital images [15]. A new gradient which takes into account the fundamental properties of the IF described in [8] and [9] is developed and adopted in order to achieve a robust estimation in a high noise environment.

II. Instantaneous Frequency Estimation

In order to introduce basic definitions of IF and WD, as well as to define the IF estimation problem, we will start the analysis by considering the signal:

$$s(t) = Ae^{j\phi(t)}. \tag{1}$$

The instantaneous frequency of this signal is defined as the first derivative of signal phase:

$$\omega(t) = \phi'(t). \tag{2}$$

In the further analysis it will be assumed that the considered signal is corrupted with additive, white, complex Gaussian noise $\varepsilon(n)$:

$$x(t) = s(t) + \varepsilon(t), \tag{3}$$

with variance σ^2. If $x(t)$ is sampled with a sampling interval Δt, the Pseudo-Wigner distribution (PWD) of such discrete-time signal is defined as:

$$PWD(t,\omega) = \sum_{k=-\infty}^{\infty} w_h(k\Delta t)x(t+k\Delta t)x^*(t-k\Delta t)e^{-j2\omega k\Delta t}, \tag{4}$$

where w_h denotes the window function of the width *h*. For numerical realizations, the discrete PWD with window function of length *N* is defined as follows:

$$PWD(n,k)=\sum_{m=-N/2}^{N/2-1} w(m/2)x(n+m/2)x^{*}(n-m/2)e^{-4\pi mk/N} \quad (5)$$

As it is emphasized in the Introduction, the estimation of the instantaneous frequency based on maxima of time-frequency representations is an often used approach. If the discrete-time PWD given with (4) is used, the IF estimation problem can be written in the following form:

$$\widehat{\omega}(t)=\arg[\max_{\omega} PWD(t,\omega)], \quad (6)$$

with $\omega(t)$ being the discrete instantaneous frequency, and $\widehat{\omega}(t)$ being its estimate. In the analyzed framework the discrete PWD is used, and thus the equivalent formulation of (6) is given with:

$$\hat{k}(n)=\arg[\max_{k} PWD(n,k)]. \quad (7)$$

The optimization problem (6), i.e. (7) is the starting point of the topic analyzed in this paper. The time-frequency plane of (5) can be considered as a discrete grid, with *n* and *k* being the discrete time and frequency indices, respectively. In the further analysis, the absolute value of the representation should be calculated. Thus, the absolute PWD can be treated as a digital image in the context of the analyzed problem. The idea behind this study is to adopt the Ant colony optimization algorithm for edge detection for the purpose of IF estimation in the presence of high-level noise, since in that case the PWD maxima are allocated from the IF frequency points, thus producing the wrong estimation when solving (6), i.e. (7).

III. Ant colony optimization algorithm for edge detection applied on the Pseudo-Wigner distribution

A digital image, as well as the absolute value of the discrete PWD, i.e. a matrix **a** with dimensions $M\times N$ can be observed as a rectangular grid, on which artificial ants move to adjacent cells. At the beginning of the optimization algorithm, a certain number of these intelligent agents are placed on the grid at random positions. In the all following iterations all the ants move following certain rules. Every agent can move only to adjacent cells (discrete points of digital image or a TFR), reinforcing a certain level of pheromone on that spot. One cell can be occupied by one ant, and ants do not move if they are totally surrounded by other ants. An iteration ends when all the ants move to adjacent cells (except the totally surrounded ones), and at the end of the iteration a certain constant level of pheromone ξ evaporates from each cell. The ants communicate via the crucial concept of pheromone deposition and evaporation, which actually represents a positive and negative feedback of the system. If the *n* and *k* are the coordinates of considered point in the lattice, its pheromone level $\sigma(n,k)$ is kept as $S(n,k)$ in a matrix **S** denoted as the pheromone map [15], [16]. Thus, the pheromone deposition and evaporation is modeled by changing the appropriate values of the matrix **S**. Note that the ants positions and orientations are also placed in an auxiliary matrix. Ant's movement and the pheromone deposition mechanisms are crucial for the control of the mass behavior of artificial ants. Every ant choses in which cell to move based on two criteria: the pheromone level in the adjacent cells and its current orientation. Since the discrete rectangular grid is considered, an ant at the position (*n*, *k*) can move to its eight adjacent cells. If the digital grid, i.e. digital image or the absolute PWD value is denoted with $a(n,k)$, then the 3×3 adjacency of the ant can be represented with the following matrix:

$$\mathbf{A}(n,k)=\begin{bmatrix} a(n-1,k-1) & a(n-1,k) & a(n-1,k+1) \\ a(n,k-1) & a(n,k) & a(n,k+1) \\ a(n+1,k-1) & a(n+1,k) & a(n+1,k+1) \end{bmatrix} \quad (8)$$

An ant at the position (*n*, *k*) has a certain orientation. At the beginning of the optimization algorithm this orientation is random for all ants. In the all following iterations the orientation of an ant is determined by its movement from the previous iteration. For example, if an ant in previous iteration is moved from the position $(n+1,k)$ to the position (n,k), then it is oriented upwards.

Another important parameter for the ant colony behavior control is $w(\Delta_\Theta)$ which influence how an ant choses to move based on its orientation. It is defined as a function of angles Δ_Θ between the current direction of an ant, and the adjacent cells positions. Since the discrete adjacency such as (8) is considered, the function $w(\Delta_\Theta)$ is defined with:

$$w(\Delta_\Theta)=\begin{cases} 1, & \Delta_\Theta=0^\circ \\ 1/2, & \Delta_\Theta=\pm 45^\circ \\ 1/4, & \Delta_\Theta=\pm 90^\circ \\ 1/12, & \Delta_\Theta=\pm 135^\circ \\ 1/20, & \Delta_\Theta=180^\circ \end{cases} \quad (9)$$

The other parameter on which depends the movement choice of an ant is the dependence on the pheromone level $\sigma(n,k)$, given by a function defined as [15], [16]:

$$W(\sigma(n,k))=\left(1+\frac{\sigma(n,k)}{1+\delta\sigma(n,k)}\right)^{\beta} \quad (10)$$

A large value of the parameter β results in ants heavily attracted with the pheromone level and vice versa. The parameter δ describes the decrease of the ants' sensitivity on the pheromone when it is highly concentrated. The probability that an ant will move from the cell *z*, which in (8) is denoted as the position (*n*, *k*), to an adjacent position *i* which corresponds to the one of its eight adjacent positions is given with [15]:

$$P_{iz}=\frac{W(\sigma_i)w(\Delta_\Theta)}{\sum_{j/z} W(\sigma_j)w(\Delta_\Theta)}, \quad (11)$$

where j/z denotes the summation over all points adjacent to $z=(n,k)$, and $i,j\in\{(n-i_1,k-i_2)\}$, with $i_1,i_2=0,\pm 1$.

When an ant moves to a point (*n*, *k*), the pheromone level in matrix **S** changes according to:

$$\sigma(n,k)=\sigma(n,k)+\upsilon+\mu\nabla(n,k)/M_a, \quad (12)$$

with M_a being the maximal value of the matrix **a**, υ is a constant predefined level of pheromone, μ is a positive step constant and $\nabla(n,k)$ denotes the gradient, i.e. a dynamic value of pheromone which is added at the position (*n, k*) visited by the agent. For the problem of IF estimation, gradient $\nabla(n,k)$ is defined taking into account the specific nature of the considered problem. It is analyzed in the following section. The parameters ξ, μ, υ, β and δ are set as in [16]. Numerical results (and the convergence of the algorithm) have shown that the satisfactory results are obtained after $I_{\max}=20$ number of iterations. Previously described algorithm can be summarized with the following steps:

Step 0: Place the intelligent agents on random positions, with random orientations on the discrete grid. Then, for 1 to $I_{\max}$ repeat steps 1-3:

Step 1: For every agent compute (11), based on (9) and (10) and move the agent to an adjacent cell, not occupied by other ants, with the probability (11).

Step 2: For every grid point (*n, k*) visited by the agent in the Step 1 update the matrix **S** using the relation (12).

Step 3: Decrease the pheromone level in the whole matrix **S** with a constant level ξ.

During the iterations, due to the gradient influence on pheromone level (12) the pheromone map **S** will obtain the largest values at the IF points. The IF is finally obtained as:

$$\hat{k}(n)=\arg[\max_k S(n,k)]\,. \tag{13}$$

In order to suppress eventually obtained impulses at some points *n*, a median filter of length 3-5 might be applied on the estimate (13).

IV. THE PROPOSED GRADIENT DEFINITION

For the gradient definition the nature of the considered problem is taken into account. Namely, although the PWD maxima in a high noise environment are allocated from the signal's IF positions with a certain probability [7], at each observed time instant *n* one of the largest PWD values will still be positioned at the IF. On the other side, it is known that IF variation between two consecutive time instants should not be too fast, which is the most common case in real scenarios [8]. Taking into account these two facts, a new gradient $\nabla(n,k)$ which defines the dynamic value of the pheromone added in (12) is defined as follows:

$$\nabla(n,k)=\Psi(\mathbf{A}(n,k))\Phi(\mathbf{A}(n,k))\Lambda(\mathbf{A}(n,k))\,. \tag{14}$$

The matrix $\mathbf{A}(n,k)$ is defined in (8). Its elements might be denoted with $A_{ij}(n,k)\in\mathbf{A}(n,k)$, with $i,j=1,2,3$. The function $\Psi(\mathbf{A}(n,k))$ is defined with:

$$\Psi(\mathbf{A}(n,k))=\prod_j\sum_i A_{ij}(n,k)/3\,. \tag{15}$$

The (15) is defined as the product of the mean values of three columns of the matrix $\mathbf{A}(n,k)$. If the auto-terms appear within the matrix $\mathbf{A}(n,k)$, then the large value of (15) is expected, since the auto-terms would appear in all three columns with a high probability. If the matrix $\mathbf{A}(n,k)$ contains only noisy PWD values, then (15) has a small value, since the noisy points are usually isolated in the time-frequency plane.

The function $\Phi(\mathbf{A}(n,k))$ represents the median of the mean values of the columns of the matrix $\mathbf{A}(n,k)$:

$$\Phi(\mathbf{A}(n,k))=\operatorname*{median}_j\left(\sum_i A_{ij}/3\right) \tag{16}$$

It is expected that this function has a small value for noisy PWD points, since they are usually isolated, not expected to appear in all three columns. The largest values of this function are expected for auto-terms, since they will appear in the all adjacent columns of the matrix $\mathbf{A}(n,k)$. Signals with IF variations which are not too fast are considered. If these variations were faster, this function could be modified in order to follow them, with the cost of higher numerical complexity. Additionally, if the signal is not ideally concentrated, even the whole matrix $\mathbf{A}(n,k)$ may contain auto-term values, thus producing a large value of (16).

Finally, the function $\Lambda(\mathbf{A}(n,k))$ takes into account the fact that IF has small variations in consecutive time instants *n*:

$$\Lambda(\mathbf{A}(n,k))=\max\left(\left[\prod\nolimits_{i=j}A_{ij}(n,k)\quad \prod\nolimits_{i=j}\tilde{A}_{ij}(n,k)\quad \mathbf{P}_i\right]\right) \tag{17}$$

The matrix with elements $\tilde{A}_{ij}(n,k)$ is obtained by reversing the column order of the original matrix **A**(*n, k*). Previous heuristic function calculates the maximal value of the vector consisted of products of elements on the diagonals of the matrix $\mathbf{A}(n,k)$ and products of the values in the first, second and the third row of this matrix. These row products are defined with:

$$\mathbf{P}_i=\left[\prod_j A_{1j}(n,k)\quad \prod_j A_{2j}(n,k)\quad \prod_j A_{3j}(n,k)\right] \tag{18}$$

If the PWD values corresponding to the IF are within the matrix $\mathbf{A}(n,k)$, then one of these products will have a large value. On the other side, isolated one or two points with large value corresponding to the noise will probably produce a small value in all products of the values both on diagonals or in rows of the matrix $\mathbf{A}(n,k)$. PWD points outside of the IF regions will produce small values of this heuristic function.

V. NUMERICAL EXAMPLES

In our numerical examples two cases are considered. In the first experiment linear frequency modulated signal (LFM) is considered, defined with:

$$x(t)=\exp(j16\pi t^2) \tag{19}$$

In the second experiment a sinusoidally modulated signal:

$$x(t)=\exp(j2\pi\sin(-3\pi t)-j25.6t)\,. \tag{20}$$

is considered. Both signals are observed for $0\le t\le 1$ and sampled with the step $\Delta t=1/128$.

The discrete IF frequency ω(*n*) was estimated using the PWD maxima approach (7) and the proposed approach, calculating (13). The signals were observed in different signal-to-noise levels (SNRs), and the estimation results are shown on the Fig. 1 for LFM, and on Fig.2 for the sinusoidally modulated signal, with SNRs: (a) 0 dB, (b) -1dB, (c) -2dB and (d) -3dB on both figures. The dotted line represents the IF estimate obtained using the PWD maxima approach, the thick line represents the estimate obtained using the proposed approach, while the solid line is the original IF.

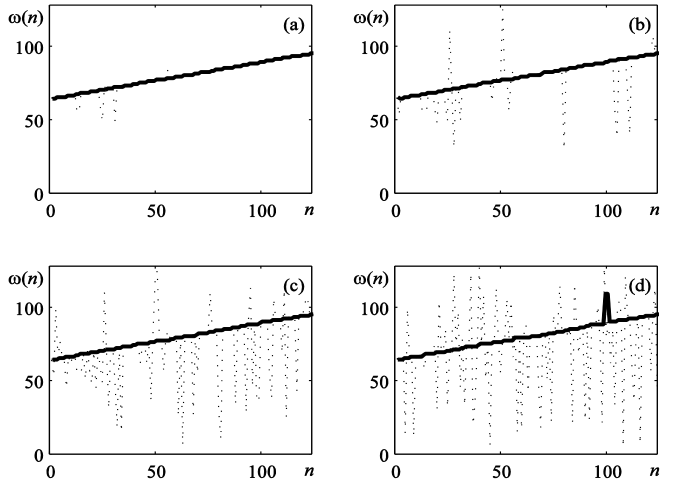

Figure 1. Estimation of the instantaneous frequency of a noisy LFM signal with: (a) SNR=0dB, (a) SNR=-1dB, (c) SNR=-2dB and (c) SNR=-3dB

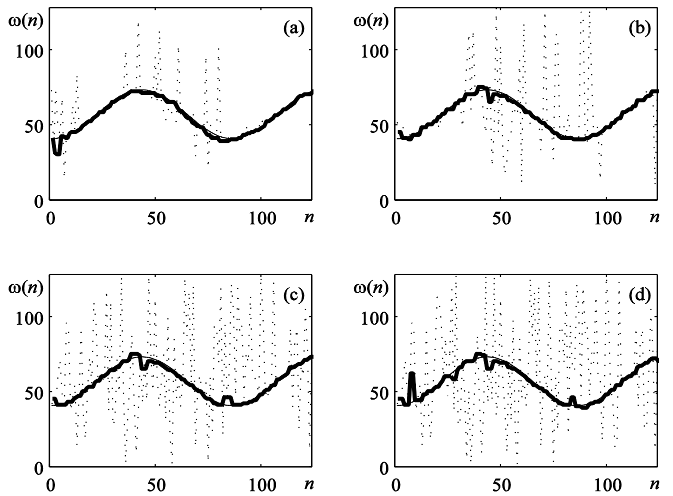

Figure 2. IF estimation of a noisy sinusoidally modulated signal with: (a) SNR=0dB, (a) SNR=-1dB, (c) SNR=-2dB and (c) SNR=-3 dB

VI. CONCLUSION

A modified version of the Ant colony optimization algorithm used for the edge detection in digital images is applied for the IF estimation of signals corrupted with a high level Gaussian noise. To this aim, a modified gradient for the pheromone level update is developed based on fundamental IF properties. The proposed algorithm successfully estimates the IF of signals whose variations which are not too fast. This kind of IF variation is common in real scenarios. However, if IF variations were faster, the heuristic functions associated to the proposed algorithm could be modified in order to follow them, with the cost of higher numerical complexity. Performances of the proposed algorithm are illustrated through numerical examples. Since the optimization problems arise in sensor networks, our future work will include the research of possible applications in this problem framework.

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