On the STFT Inversion Redundancy

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***Abstract*—Inversion of the short-time Fourier transform (STFT) calculated with common windows is revisited. It has been shown that using commonly windowed STFT a full signal reconstruction may be achieved from the same number of the STFT coefficients as the number of signal samples. Architectures for the redundant and nonredundant STFT realization are presented. Inversion stability is checked through a noise analysis in the reconstruction process.**

I. Introduction

Most commonly used tool in time-frequency analysis is the short-time Fourier transform (STFT) [1]-[13]. Significant research efforts have been made to provide efficient signal reconstruction schemes from the STFT, [1], [7]. The most significant drawback of the STFT is its redundancy. A simple nonredundant signal reconstruction from the STFT is presented in this paper, along with a noise analysis confirming the inversion stability.

II. Definitions

The basic idea behind the STFT is to apply the Fourier transform to a localized signal $x(n)$, obtained by using a sliding window function $w(m)$. Discrete time-frequency domain form of the STFT, at an instant n and frequency k, reads

$$S_N(n,k)=\sum_{m=-N/2}^{N/2-1} x(n+m)w(m)e^{-j\frac{2\pi}{N}mk}.$$

The STFT $S_N(n,k)$ is calculated using signal samples within a window of the width N. Assuming a rectangular window for a given time instant n_i we can write

$$\mathbf{S}_N(n_i)=\mathbf{W}_N\mathbf{X}_N(n_i), \quad (1)$$

where $\mathbf{S}_N(n_i)$ and $\mathbf{X}_N(n_i)$ are column vectors with elements $S_N(n_i,k)$, $k=-N/2$,..., $N/2-1$ and $x(n_i+m)$, $m=-N/2$,..., $N/2-1$, respectively. Matrix $\mathbf{W}_N$ is an $N\times N$ DFT matrix with elements $\exp(-j2\pi mk/N)$, where m is the column index and k is the row index of the matrix. A matrix form for all STFT nonoverlapping values is

$$\mathbf{S}_M=\begin{bmatrix} \mathbf{W}_N & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{W}_N & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{W}_N \end{bmatrix}\mathbf{X}=\tilde{\mathbf{W}}_{M,N}\mathbf{X}, \quad (2)$$

where $\mathbf{S}_M$ is a column vector containing all STFT vectors $\mathbf{S}_N(n_i)$, at $n_i=N/2,3N/2$,..., $M-N/2$, and $\mathbf{X}$ is a column vector with all signal samples

$$\mathbf{X}=[x(0),x(1),...,x(M-1)]^T,$$

while $\tilde{\mathbf{W}}_{M,N}$ is a block $M\times M$ matrix formed from the smaller DFT matrices $\mathbf{W}_N$, as in (2).

Rectangular windows have poor localization in the frequency domain. Study of well-localized window forms in the time-frequency domain has been an important topic, since the STFT concept was introduced. In general, for a nonrectangular window, definition (1) is modified as

$$\mathbf{S}_N(n_i)=\mathbf{W}_N\mathbf{H}_N\mathbf{X}_N(n_i),$$

where $\mathbf{H}_N$ is a diagonal $N\times N$ matrix with the window values on the diagonal, $H_N(m,m)=w(m)$, $m=-N/2,...,N/2-1$. In a matrix notation, for the nonoverlapping case, we get

$$\mathbf{S}_M=\tilde{\mathbf{W}}_{M,N}\tilde{\mathbf{H}}_{M,N}\mathbf{X}, \quad (3)$$

where $\tilde{\mathbf{W}}_{M,N}$ and $\tilde{\mathbf{H}}_{M,N}$ are $M\times M$ matrices formed from smaller $N\times N$ matrices $\mathbf{W}_N$ and $\mathbf{H}_N$, respectively, as in (2).

Nonoverlapping cases are important and easy for analysis. They also keep the number of the STFT coefficients equal to the number of signal samples. However, in the case of nonrectangular windows some of the signal samples are weighted by very small numbers. This is undesired in the signal inversion.

III. Inversion

The inversion of a nonoverlapping STFT with a rectangular window, defined by (1) or (2), is straightforward using either $\mathbf{X}_N(n_i)=\mathbf{W}_N^{-1}\mathbf{S}_N(n_i)$, or in full matrix form $\mathbf{X}=\tilde{\mathbf{W}}_{M,N}^{-1}\mathbf{S}_M$. Inversion for a nonrectangular window is

$$\tilde{\mathbf{H}}_{M,N}\mathbf{X}=\tilde{\mathbf{W}}_{M,N}^{-1}\mathbf{S}_M. \quad (4)$$

Problem in this inversion are small values in $\tilde{\mathbf{H}}_{M,N}$. If there is any disturbance in the calculated STFT it will be present in the product $\tilde{\mathbf{H}}_{M,N}\mathbf{X}$. The reconstruction of signal $\mathbf{X}$, after division of $\tilde{\mathbf{H}}_{M,N}\mathbf{X}$ by small values in $\tilde{\mathbf{H}}_{M,N}$ will be extremely unreliable and unacceptable.

A common way to avoid this problem is to use redundant STFT calculation, with overlap-add method. In the simplest and the least redundant case with common windows, an overlapping of $N/2$ is used. The window function is chosen such that $w(m)+w(m-N/2)=const.$ for the interval where the windows overlap. This property is satisfied by the Hann, Hamming, Bartlett, and Blackman window[1]. Then, in

[1]The number of input signal values is M. A nonredundant STFT cannot contain less than M independent values. The property that a sum of overlapping windows is constant holds for the common windows with time steps $N/2$, $N/4$, $N/8$, and so on. With steps smaller than $N/2$ the redundancy is additionally increased. With an increase of downsampling factor in the frequency domain good frequency coverage would be lost, Fig.2.

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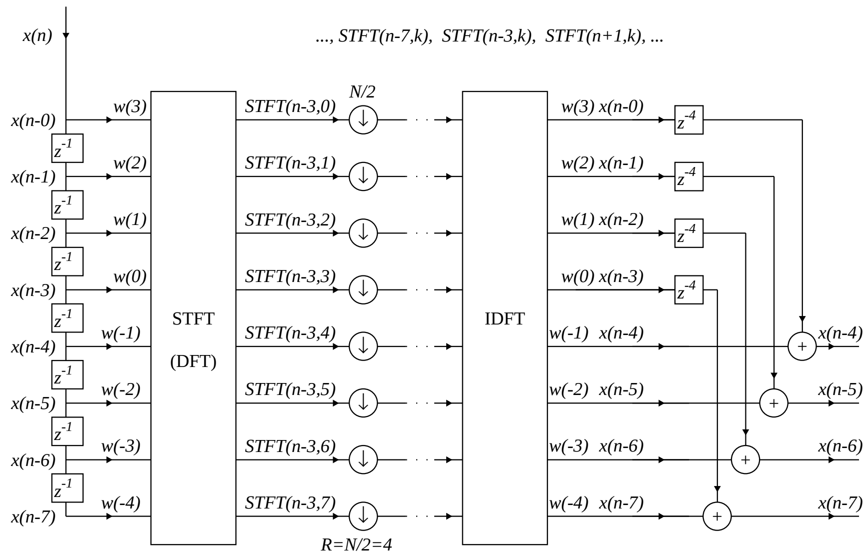

Fig. 1. System for a redundant realization of the STFT transform based on overlap and add method. Illustration for $N = 8$ with $STFT(n,k) = STFT(n,k+N)$.

addition to the nonoverlapped STFT, calculated at $n_i = N/2$, $3N/2$,...,$M - N/2$ according to (3), another set of STFT values, overlapping with the previous one, is calculated at $n_i = 0$, N, ..., M. Its values are

$$\mathbf{S}_{M+N} = \tilde{\mathbf{W}}_{M+N,N}\tilde{\mathbf{H}}_{M+N,N}[\mathbf{0}_{N/2}\ \mathbf{X}^T\ \mathbf{0}_{N/2}]^T, \quad (5)$$

where $\mathbf{0}_{N/2}$ is a row vector with $N/2$ zeros as its elements, $\tilde{\mathbf{W}}_{M+N,N}$ is an $(M+N)\times(M+N)$ matrix formed using $\mathbf{W}_N$ matrix as in (2), and $\tilde{\mathbf{H}}_{M+N,N}$ is the corresponding diagonal window matrix, with $M+N$ windows $w(m)$ on the diagonal. Notation $\mathbf{X}_Z$ will be used for the zero-padded signal,

$$\mathbf{X}_Z = [\mathbf{0}_{N/2}\ \mathbf{X}^T\ \mathbf{0}_{N/2}]^T.$$

The inversion of this STFT produces

$$\tilde{\mathbf{H}}_{M+N,N}\mathbf{X}_Z = \tilde{\mathbf{W}}^{-1}_{M+N,N}\mathbf{S}_{M+N}. \quad (6)$$

A part of $\tilde{\mathbf{H}}_{M+N,N}\mathbf{X}_Z$ from the previous relation, after the added zero values at the beginning and ending intervals (needed to produce an $N/2$ shift in the calculation) are removed, will be denoted by $\left\lceil\tilde{\mathbf{H}}_{M+N,N}\mathbf{X}_Z\right\rfloor_M$. Now this vector is reduced to the same number of samples as $\mathbf{X}$. If we sum $\tilde{\mathbf{H}}_{M,N}\mathbf{X}$ from (4) and $\left\lceil\tilde{\mathbf{H}}_{M+N,N}\mathbf{X}_Z\right\rfloor_M$, with the assumption that the window is such that a sum of its values $w(m)$ with their shifted versions $w(m - N/2)$ is constant (without loss of generality assume that this sum is equal to 1), we have

$$\tilde{\mathbf{H}}_{M,N}\mathbf{X}+\left\lceil\tilde{\mathbf{H}}_{M+N,N}\mathbf{X}_Z\right\rfloor_M = \mathbf{X},$$

resulting in

$$\mathbf{X} = \tilde{\mathbf{W}}^{-1}_{M,N}\mathbf{S}_M+\left\lceil\tilde{\mathbf{W}}^{-1}_{M+N,N}\mathbf{S}_{M+N}\right\rfloor_M.$$

In this case the inversion is achieved from the STFT obtained from (4) and overlapping STFT from (6) without need for a division by (possibly small) window function values, Fig.1.

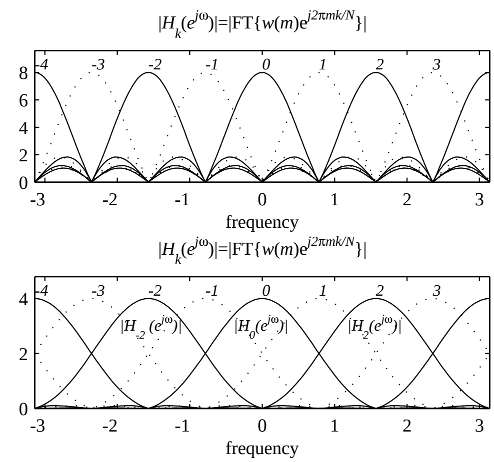

Fig. 2. Illustration of the basis (transfer) functions, representing modulated rectangular window (top) and the modulated Hann window in the STFT analysis with $N = 8$.

However redundancy of this kind of inversion is high (100%). In total an order of $2M$ STFT coefficients are used to get M signal values, if the signal is considered as circular with $N/2$ first and $N/2$ last signal samples being used for one STFT calculation. Otherwise the number of STFT coefficients is $2M + N$.

Here we will present a method to reduce/avoid the redundancy in the STFT calculation. Since the STFT is calculated with the frequency smoothing overlapping windows (corresponding basis functions are presented in Fig.2) we may expect that one such STFT coefficient is a good frequency represent of two frequency points, Fig.2(bottom), in contrast to the rectangular based basis functions, Fig.2(top). Then the STFT can be calculated only at the frequencies $2k$, as

$$S_N(n,2k)=\sum\nolimits_{m=-N/2}^{N/2-1}x(n+m)w(m)e^{-j\frac{2\pi}{N}m2k}$$
$$=\sum_{m=0}^{N/2-1}[x(n+m)w(m)$$
$$+x(n+m-\frac{N}{2})w(m-\frac{N}{2})]e^{-j\frac{2\pi}{N/2}mk}. \quad (7)$$

In this way, the STFT is downsampled in frequency by a factor of 2. The matrix form of this relation is

$$\mathbf{STFT}=\mathbf{S}^{\downarrow 2}_{M+N/2}=\tilde{\mathbf{W}}_{M+N/2,N/2}\mathbf{X}_a \quad (8)$$

where $\mathbf{S}^{\downarrow 2}_{M+N/2}$ denotes the STFT downsampling in the frequency domain. Notation $\mathbf{STFT}$ will be used for this column vector. Elements of vector $\mathbf{STFT}$ are frequency downsampled STFT values $S_N(n,2k)$. Notation $\tilde{\mathbf{W}}_{M+N/2,N/2}$ is used for a $(M+N/2)\times(M+N/2)$ matrix formed from the DFT matrices of size $\mathbf{W}_{N/2}$, and $\mathbf{X}_a$ is a new signal formed from the original signal, with aliasing and window functions, as in (7). Elements of vector $\mathbf{X}_a$ are defined by (7)

$$x_a(n_i+m)=x(n_i+m)w(m)+x(n_i+m-\frac{N}{2})w(m-\frac{N}{2})$$
$$\text{for } n_i=0,\ N/2,\ N,...,M \text{ and } m=0,1,...,N/2-1$$
$$\text{with } x(n)=0 \text{ for } n<0 \text{ and } n\geq M. \quad (9)$$

Vector $\mathbf{X}_a$ is of duration $M+N/2$. It is important to note that each signal value $x(n)$ appears exactly twice, in two values of $x_a(n)$. For example, for $N=8$ and $M=64$ the value of $x(6)$ appears in $x_a(4+2)=x(6)w(2)+x(2)w(-2)$ and in $x_a(8+2)=x(10)w(2)+x(6)w(-2)$. Value $x(6)$ appears with weighting coefficient $w(2)$ and with $w(2-N/2)=w(-2)$. Since $w(2)+w(2-N/2)=1$ it means that we will always have at least one equation with a significant contribution of $x(6)$.

The matrix form of signal transformation (9) is $\mathbf{X}_a=\mathbf{TX}$. For example, for $M=8$ and $N=4$ for the Hann window (with elements: $w_m=w(m)=0,\ 1/2,\ 1,\ 1/2$ for $m=-2$, $-1,\ 0,\ 1$, respectively), the matrix

$$\mathbf{T}=\begin{bmatrix} w_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & w_1 & 0 & 0 & 0 & 0 & 0 & 0\\ w_{-2} & 0 & w_0 & 0 & 0 & 0 & 0 & 0\\ 0 & w_{-1} & 0 & w_1 & 0 & 0 & 0 & 0\\ 0 & 0 & w_{-2} & 0 & w_0 & 0 & 0 & 0\\ 0 & 0 & 0 & w_{-1} & 0 & w_1 & 0 & 0\\ 0 & 0 & 0 & 0 & w_{-2} & 0 & w_0 & 0\\ 0 & 0 & 0 & 0 & 0 & w_{-1} & 0 & w_1\\ 0 & 0 & 0 & 0 & 0 & 0 & w_{-2} & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & w_{-1} \end{bmatrix} \quad (10)$$

is invertible in the least squared sense ($\mathbf{T}^H\mathbf{T}$ is invertible) in contrast to the diagonal form corresponding to the nonoverlapping calculation with the Hann window,

$$\mathbf{T}=\operatorname{diag}(\left[0,\ \frac{1}{2},\ 1,\ \frac{1}{2},0,\ \frac{1}{2},\ 1,\ \frac{1}{2}\right]).$$

Here we used the Hann window to emphasize small (here exactly zero) values in the window.

A common assumption that the signal $x(n)$ is periodic with M would result in nonredundant calculation. In this case (9) would be calculated as

$$\text{for } n_i=0,\ N/2,\ N,\ ...,M \text{ and } m=0,1,...,N/2-1$$
$$\text{with } x(n)=x(n+M) \text{ for } n<0. \quad (11)$$

For $N/2=2$ last two rows in (10) would be just omitted and their values added to the first $N/2=2$ rows, $T(1,7)=w_{-2}$ and $T(2,8)=w_{-1}$. However, this matrix would be singular, loosing the invertibility property. Therefore, with a simple periodic signal assumption, redundancy free calculation of the STFT is not possible.

It is possible to achieve a redundancy free STFT with the assumption of periodic $x(n)$ using a change in the calculation, in order to avoid matrix singularity. The coefficients $S_N(n,2k-1)$ are used instead of $S_N(n,2k)$ for the STFT part corresponding to the signal periodic extension. Note that $S_N(n,2k-1)$ is just a value of $S_N(n,2k)$ for a signal modulated by $\exp(j2\pi m/N)$. It means that $x(n_i+m-N/2)$ in (9) is multiplied by $\exp(j2\pi m/N)$ for $n=n_i+m-N/2<0$, when $x(n+M)$ is used instead of $x(n)$ in $S_N(n,2k)$,

$$x(n_i+m-N/2)=x(M+(n_i+m-N/2))e^{j2\pi m/N} \quad (12)$$

for $n_i+m-N/2<0$ in (11). The STFT is calculated for this new signal, denoted by $\mathbf{X}_{ap}$, as

$$\mathbf{STFT}=\tilde{\mathbf{W}}_{M,N/2}\mathbf{X}_{ap}. \quad (13)$$

Change in the example matrix $\mathbf{T}$ in (10) is such that the last $N/2$ rows are omitted and added to the first $N/2$ rows. Then $T(1,7)=-w_{-2}$ and $T(2,8)=-jw_{-1}$. The reconstruction results are the same as in the slightly redundant case (9) and (10). Matrix $\mathbf{T}$ is now complex.

The reconstructed signal $\mathbf{X}_R$ is obtained either from (13) or (8) with $\mathbf{X}_{ap}=\mathbf{TX}_R$ or $\mathbf{X}_a=\mathbf{TX}_R$ as

$$\mathbf{TX}_R=\mathbf{X}_a=\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}\mathbf{STFT}$$
$$\mathbf{T}^H\mathbf{TX}_R=\mathbf{T}^H\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}\mathbf{STFT}$$
$$\mathbf{X}_R=\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}\mathbf{STFT}. \quad (14)$$

Redundancy in (14) is, for example, for $M=512$ and $N=32$ only 3%. This form will be referred to as almost nonredundant. For a redundancy free calculation using (13) the matrices $\tilde{\mathbf{W}}^{-1}_{M,N/2}$ and corresponding $\mathbf{T}^H$ should be used with the signal $\mathbf{X}_{ap}$.

Forms of (9) and (10) indicate a possibility of recursive signal reconstruction for any window, instead of the matrix $\mathbf{T}^H\mathbf{T}$ inversion. Matrix $\mathbf{T}^H\mathbf{T}$ is a tridiagonal matrix with elements $w^2(m)+w^2(m-N/2)$ on the diagonal, periodically extended up to $M+N/2$. Subdiagonal and updiagonal have the elements $w(m)w(m-N/2)$, Fig.3. Note that the matrix $\mathbf{T}^H\mathbf{T}$ inversion is signal independent. Solving a linear system of equations with a tridiagonal matrix has recently been a hot topic in mathematics, with several solutions proposed to get the result without matrix inversion calculation [14],[15]. In the presented simulations we have used a direct inversion.

Within the reconstruction system framework the STFT inverse vector $\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}\mathbf{STFT}$ in (14) can be considered as an input $y(n)$ to M FIR systems, whose impulse

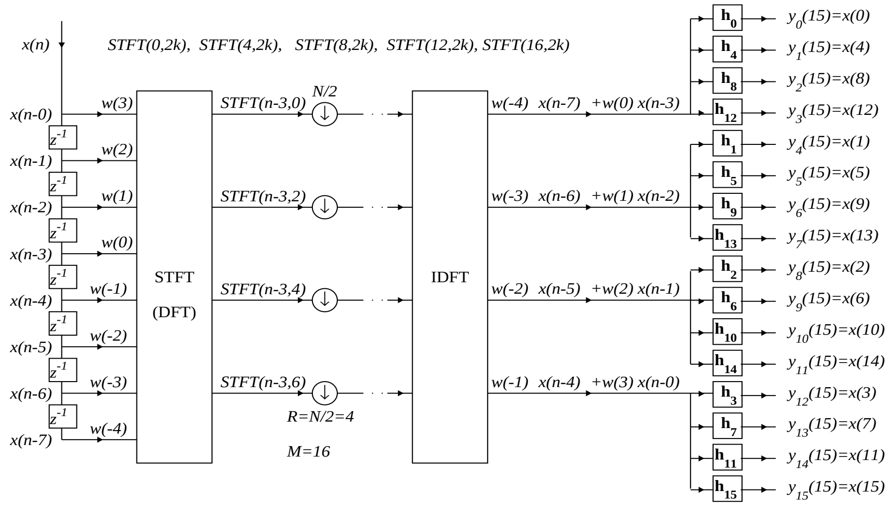

Fig. 4. System for an almost nonredundant realization of the STFT transform. Illustration for $M=16$ and $N=8$.

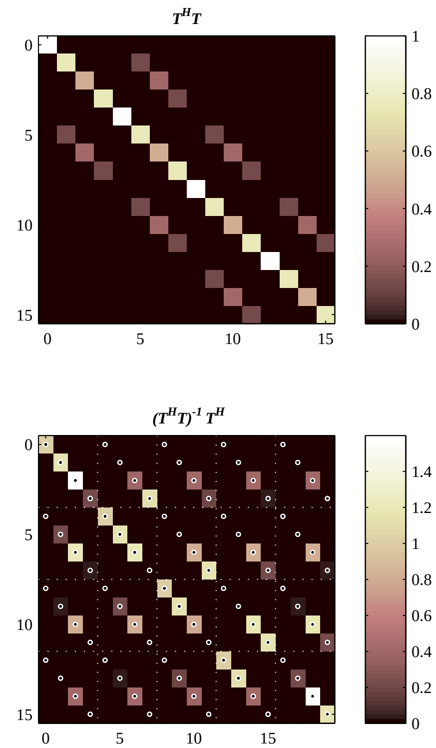

Fig. 3. Tridiagonal matrix $\mathbf{T}^H\mathbf{T}$ and transformation matrix $\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H$, defining reconstruction coefficient filters $\mathbf{h}_i$. Absolute values are presented.

responses are $\mathbf{h}_i$, $i = 0,1,2...,M-1$. Elements of $\mathbf{h}_i$ are corresponding values of matrix $\left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H$ rows in each block of size $N/2 = 4$ (dotted lines in Fig.3(b)). For the presented example $\mathbf{h}_0 = (1,0,0,0,0)$, $\mathbf{h}_1 = (1.17,0,0,0.0002,0.001)$, $\mathbf{h}_2 = (1.6,0.4,-0.4,0.4,-0.4)$, $\mathbf{h}_3 = (0.20,1.13,-0.20,0.03,-0.006)$, $\mathbf{h}_4 = (0,1,0,0,0)$, and so on. Then the output of these systems at instant $n = M-1$ will be the values of signal $x(i)$, Fig.4.

IV. Noise (Error) Analysis

The inversion stability will be checked on the sensitivity to noise (error) in the inversion. Assume that the values of

$$\mathbf{STFT} = \mathbf{S}^{\downarrow 2}_{M+N/2,N/2}$$

contain a small noise $\mathbf{STFT}+\varepsilon$ introduced after the STFT calculation. If the STFT inversion uses any division with a small number or any low conditioned matrix, the output noise will be highly amplified and the signal to noise ratio will be significantly degraded. The reconstructed signal for noisy coefficients $\mathbf{STFT}+\varepsilon$ is, (14),

$$\mathbf{X}_R = \left(\mathbf{T}^H\mathbf{T}\right)^{-1}\mathbf{T}^H\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}(\mathbf{STFT}+\varepsilon) = \mathbf{X}+\varepsilon_R,$$

where $\varepsilon_R = \left(\mathbf{T^H T}\right)^{-\mathbf{1}}\mathbf{T^H}\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}\varepsilon$. Input noise is white with variance σ^2_ε. Variance of the output noise is

$$\sigma^2_{\varepsilon_R} = \mathrm{E}\{\varepsilon_R^H\varepsilon_R\} = \mathrm{E}\{\varepsilon^H\mathbf{P}^H\mathbf{P}\varepsilon\} = \sigma^2_\varepsilon\sum_{i=1}^{M+N/2}\lambda_i \quad (15)$$

where $\mathbf{P} = \left(\mathbf{T^H T}\right)^{-\mathbf{1}}\mathbf{T^H}\tilde{\mathbf{W}}^{-1}_{M+N/2,N/2}$ and Λ is a matrix of the eigenvalues λ_i of the matrix $\mathbf{P}^H\mathbf{P}$. Similar relations hold for the redundancy free calculation.

Example 1: Random signal with M samples is used as an input signal $x(n)$. Its downsampled STFT is calculated with an N sample Hann window using (8). The signal is reconstructed using three presented methods. A Gaussian noise with $\sigma_\varepsilon = 10^{-5}$ is added to the STFT before reconstruction. The standard deviation of error in the reconstructed signal $\hat{\sigma}_{\varepsilon_R}$, averaged over 100 signal realizations, is presented in following Table:

$M =$	64	256	1024
$N =$	16	32	64
$\hat{R}_1 =$	0.5299	0.3959	0.2991
$\hat{R}_2 =$	0.4221	0.3357	0.2726
$\hat{R}_3 =$	0.2165	0.1531	0.1083
$R_T =$	0.5235	0.3964	0.3002

Ratio of input and reconstructed signal noise $\hat{R} = \hat{\sigma}_{\varepsilon_R}/\sigma_\varepsilon$ is presented for the almost nonredundant STFT calculation

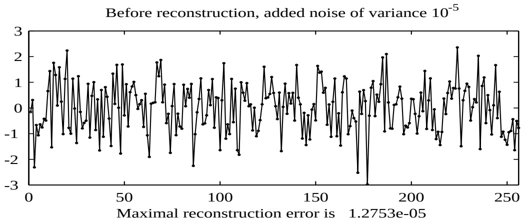

Fig. 5. Reconstruction of a signal whose duration is $M = 256$, using the same total number of 256 STFT coefficients, calculated with a Hann window with $N = 32$. Signal is reconstructed after a noise is added to the STFT values.

$\hat{R}_1$, nonredundant STFT $\hat{R}_2$, and redundant STFT with 100% redundancy factor, $\hat{R}_3$. The theoretical results for almost nonredundant STFT using (15) are denoted by R_T. All these values are of the same order. As expected the standard deviation is the smallest in the case of redundant calculation. Reconstruction result for one realization and redundancy free STFT is shown in Fig.5. Noise with $\sigma_\varepsilon = 10^{-5}$ is added. The maximal error in 512 samples is $1.2753 \cdot 10^{-5}$. It is of an $3\sigma_{\varepsilon_R} = 1.3461 \cdot 10^{-5}$ order, as expected from three-sigma rule for the Gaussian distribution.

Example 2: The redundant STFT with all frequency samples and the STFT with reduced frequency samples are calculated for a sum of real-valued sinusoid, linear frequency modulated signal and two chirps with $M = 1024$ samples and a Hann window with $N = 64$. The STFTs are shown in Fig. 6(a), (b). Time-frequency concentration is compared using the norm-one based measure, [16]. The measure values, normalized to the whole time-frequency range in each case, are $\|\mathbf{STFT}\|_1 = 2.36$, $\|\mathbf{STFT}\|_1 = 2.34$ and $\|\mathbf{STFT}\|_1 = 2.34$ for the redundant, almost nonredundant, and nonredundant STFT, respectively. They are almost the same. Using every other frequency sample the linearity property with respect to the signal is preserved along with a good coverage of the time-frequency plane. It means that filtering, by using time-frequency masks, can be done as in the case when each frequency sample is used. A noise is added to the nonredundant STFT, Fig.6(c). The STFT is then filtered by using a threshold at $0.25 \max|\mathbf{STFT}|$. The nonredundant filtered STFT is obtained, Fig.6(d). Noise overlapping with the STFT values remains. The number of these points where noise will remain is calculated for this signal in an ideal nonnoisy case. It would guarantee the SNR improvement of about 7.54 dB. In the presented case the signal is recovered from the filtered noisy noredundant STFT, Fig.6(d). The SNR is 13.63 dB. An improvement of 6.31 dB with respect to the reconstruction without filtering is achieved, since some signal samples (those bellow threshold) are lost as well.

The experiment is repeated with a noise-free STFT. Using 33% largest values, and setting the other values in the nonredundant STFT to 0, produces a signal with reconstruction error -41.24 dB. Thus in addition to the fact that an exact recovery with a nonredundant STFT is possible, it allows a significant compression ratio with a small reconstruction error. Signal with $N = 1024$ samples is recovered using 341 STFT values.

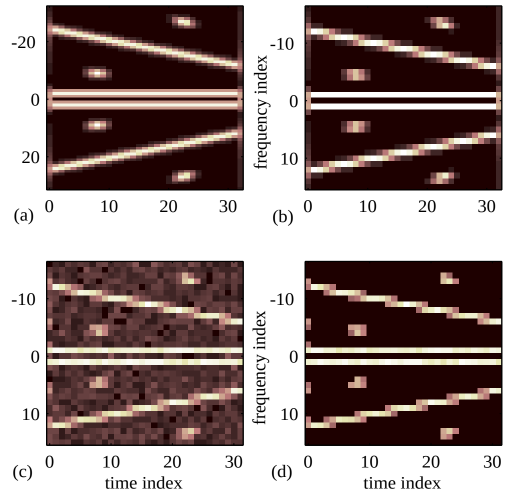

Fig. 6. (a) The STFT with all frequency samples of sum of real-valued sinusoid, linear frequency modulated signal and two chirps. (b) The STFT with reduced frequency grid. (c) Nonredundant STFT of noisy signal. (d) Nonredundant filtered STFT of noisy signal.

V. Conclusion

A scheme for a full signal reconstruction from the STFT calculated with a common window, without redundancy is presented. The reconstruction stability is demonstrated on the analysis of noise in the reconstruction process.

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