Compressive sensing approach in the Hermite transform domain

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Abstract— Compressive sensing has attracted significant interest of researchers providing an alternative way to sample and reconstruct the signals. This approach allows us to recover the entire signal from just a small set of random samples, whenever the signal is sparse in certain transform domain. Therefore, exploring the possibilities of using different transform basis is an important task, needed to extend the field of compressive sensing applications. In this paper, a compressive sensing approach based on the Hermite transform is proposed. The Hermite transform by itself provides compressed signal representation based on a smaller number of Hermite coefficients compared to the signal length. Here, it is shown that for a wide class of signals characterized by sparsity in the Hermite domain, accurate signal reconstruction can be achieved even if incomplete set of measurements is used. Advantages of the proposed method are demonstrated on numerical examples. The presented concept is generalized for the short-time Hermite transform and combined transform.

Index Terms—Compressive sensing, short-time Hermite transform, signal reconstruction in the combined domain

1. Introduction

The Hermite polynomials and Hermite functions have attracted the attention of researchers in various fields of engineering and signal processing [1]-[7], such as in quantum mechanics (harmonic oscillators), ultra-high band telecommunication channels, ECG data compression using Hermite functions representation of the QRS complexes, etc. A set of Hermite functions forming an orthonormal basis is suitable for approximation, classification and data compression tasks [3]. Since the Hermite functions are eigenfunctions of the Fourier transform, time and frequency spectra are simultaneously approximated. Here, we are especially interested in a class of signals that are sparse in the Hermite transform domain. Note that, generally, such signals are not sparse in the Fourier transform domain. In the light of compressive sensing (CS) theory [8]-[12], we propose the method for efficient reconstruction of signals from its incomplete set of samples using the Hermite transform. The number of Hermite functions used in this approach is much smaller compared to the original length of the signal. The proposed approach is useful in the applications were the significant information is missing and the total signal reconstruction is required. Note that the large amount of missing signal samples may occur as a consequence of the compressed sampling strategy, but also as a consequence of discarding damaged signal parts [13]-[15]. The theory is illustrated through examples showing that the Hermite transform based CS for certain types of signals can outperform the Fourier transform related reconstructions. Furthermore, in analogy with the time-frequency analysis based on the Fourier transform, the short-time Hermite transform is defined as a linear representation that reveals the local behavior of windowed signal parts. If the signal components are of short duration, then the short-time Hermite transform is more suitable for compressive sensing than the standard Hermite transform. Finally, the possibility of combing different transforms depending on the signal characteristics is explored.

The paper is organized as follows. The theory behind the Hermite transform and the fast method for Hermite coefficients calculation is given in Section II. The formulation of Compressive sensing approach in the Hermite transform domain is given in Section III. The possibilities to exploit other sparsity domains based on the short-time Hermite transform and combined transform are presented in Section IV. The experimental evaluation of the proposed approach is given in Section IV, while the concluding remarks are given in Section V.

2. HERMITE TRANSFORM

The Hermite functions provides good localization and the compact support in both time and frequency domain [4]. The i-th order Hermite function is defined as:

$$\psi_i(t)=\frac{(-1)^i e^{t^2/2}}{\sqrt{2^i i!\sqrt{\pi}}}\cdot\frac{d^i(e^{-t^2})}{dt^i}. \tag{1}$$

The Hermite functions provide an orthonormal basis set for an optimal representation of different signals using the fewest number of basis functions. Signal expansion into Hermite functions, known as the Hermite transform, has been used for both 1D and 2D signals in various applications. The Hermite expansion for a signal $f(t)$ can be defined as:

$$\hat{f}(t)=\sum_{i=0}^{N-1} C_i\psi_i(t)\ , \tag{2}$$

where $\psi_i(t)$ are the Hermite functions and N is the number of functions used for the approximation. The number of Hermite functions N could be usually much smaller than the number of signal samples M ($N \le M$). The Hermite coefficients can be defined by using the Hermite polynomials as follows:

$$C_i=\frac{1}{\sqrt{2^i i!\sqrt{\pi}}}\int_{-\infty}^{\infty} e^{-t^2}\left(f(t)e^{t^2}\right)H_i(t)dt, \tag{3}$$

where,

$$H_i(t)=(-1)^i e^{t^2}\frac{d^i(e^{-t^2})}{dt^i}\ , \tag{4}$$

represents the Hermite polynomial. An efficient procedure for calculation of Hermite coefficients, can be done by applying the Gauss-Hermite quadrature [5]:

$$C_i=\frac{1}{\sqrt{2^i i!\sqrt{\pi}}}\sum_{m=1}^{M}\frac{2^{M-1}M!\sqrt{\pi}}{M^2H_{M-1}^2(t_m)}\left(f(t_m)e^{t_m^2/2}\right)H_i(t_m), \tag{5}$$

where t_m are zeros of Hermite polynomials. By using the Hermite functions instead of polynomials, a simplified expression is obtained:

$$C_i\approx\frac{1}{M}\sum_{m=1}^{M}\mu_{M-1}^i(t_m)f(t_m). \tag{6}$$

The constants $\mu_{M-1}^i(t_m)$ are calculated as:

$$\mu_{M-1}^i(t_m)=\frac{\psi_i(t_m)}{(\psi_{M-1}(t_m))^2}\ . \tag{7}$$

3. COMPRESSIVE SENSING FORMULATION IN THE HERMITE TRANSFORM DOMAIN

Generally, the compressive sensing scenarios are focused on the new sampling strategy, which results in a large number of randomly missing samples comparing to the standard sampling methods [8]. Hence, based on a small set of acquired measurements, the entire signal needs to be reconstructed. The missing samples in compressive sensing generally cannot be recovered using standard interpolation methods due to the complexity of nonstationary signals in real applications. Namely, the interpolation methods such as polynomial fit, cubic spline interpolation, or similar usually assume certain model function, which is mostly inappropriate for time domain signal modeling. Therefore, the compressive sensing reconstruction is formulated in the literature as an optimization problem (rather than interpolation) which reconstructs the signal by finding the sparsest transform domain solution corresponding to the available small set of samples.

In the CS context, we are dealing with a small set of randomly chosen samples of $f(t)$. Let us assume that we have only M_A out of M available samples (M is the total number of signal samples and $M>>M_A$). The vector of available measurements is denoted as $\mathbf{y}$. Now we may write:

$$\mathbf{y}=\mathbf{\Phi f}\ , \tag{8}$$

where $\mathbf{f}$ is original full signal (of length M) written in the vector form, while $\mathbf{\Phi}$ ($M_A \times M$) is the measurement matrix. The original signal $\mathbf{f}$ can be expressed using the Hermite transform as follows:

$$\mathbf{f}=\mathbf{\Psi C}\ , \tag{9}$$

where $\mathbf{C}$ is the vector of Hermite transform coefficients, while the $\mathbf{\Psi}$ is the inverse Hermite transform matrix of size $M\times N$ ($N\leq M$):

$$\mathbf{\Psi}=\begin{bmatrix} \psi_0(0) & \psi_1(0) & \dots & \psi_{N-1}(0) \\ \psi_0(1) & \psi_1(1) & \dots & \psi_{N-1}(1) \\ \dots & \dots & \dots & \dots \\ \psi_0(M-1) & \psi_1(M-1) & \dots & \psi_{N-1}(M-1) \end{bmatrix},\quad \mathbf{H}=\frac{1}{N}\begin{bmatrix} \frac{\psi_0(0)}{\left(\psi_{N-1}(0)\right)^2} & \frac{\psi_0(1)}{\left(\psi_{N-1}(1)\right)^2} & \dots & \frac{\psi_0(M-1)}{\left(\psi_{N-1}(M-1)\right)^2} \\ \frac{\psi_1(0)}{\left(\psi_{N-1}(0)\right)^2} & \frac{\psi_1(1)}{\left(\psi_{N-1}(1)\right)^2} & \dots & \frac{\psi_1(M-1)}{\left(\psi_{N-1}(M-1)\right)^2} \\ \dots & \dots & \dots & \dots \\ \frac{\psi_{N-1}(0)}{\left(\psi_{N-1}(0)\right)^2} & \frac{\psi_{N-1}(1)}{\left(\psi_{N-1}(1)\right)^2} & \dots & \frac{\psi_{N-1}(M-1)}{\left(\psi_{N-1}(M-1)\right)^2} \end{bmatrix}. \tag{10}$$

The direct Hermite transform matrix is given by **H**. In the extended form, (9) can be written as follows:

$$\underbrace{\begin{bmatrix} f(0) \\ f(1) \\ \dots \\ f(M-1) \end{bmatrix}}_{\mathbf{f}} = \underbrace{\begin{bmatrix} \psi_0(0) & \psi_1(0) & \dots & \psi_{N-1}(0) \\ \psi_0(1) & \psi_1(1) & \dots & \psi_{N-1}(1) \\ \dots & \dots & \dots & \dots \\ \psi_0(M-1) & \psi_1(M-1) & \dots & \psi_{N-1}(M-1) \end{bmatrix}}_{\Psi} \underbrace{\begin{bmatrix} C(0) \\ C(1) \\ \dots \\ C(N-1) \end{bmatrix}}_{\mathbf{C}}.$$

The Hermite basis functions are calculated using the fast recursive realization defined as:

$$\begin{gathered} \psi_0(t)=\frac{1}{\sqrt[4]{\pi}}e^{-t^2/2}, \quad \psi_1(t)=\frac{\sqrt{2}t}{\sqrt[4]{\pi}}e^{-t^2/2}, \\ \psi_i(t)=t\sqrt{\frac{2}{i}}\Psi_{i-1}(t)-\sqrt{\frac{i-1}{i}}\Psi_{i-2}(t), \ \forall i\geq 2. \end{gathered} \tag{11}$$

According to (8) and (9) we have:

$$\mathbf{y}=\mathbf{\Phi\Psi C}=\mathbf{\Theta C}\,. \tag{12}$$

The reconstructed signal **f** can be obtained as a solution of M linear equations with M_A unknowns. The system is undetermined and can have infinitely many solutions. Now we assume that the signal is sparse in the Hermite transform domain. It means that the observed signal can be efficiently represented by a very small number K of Hermite expansion coefficients, such that $K<M_A$. Therefore, the optimization based mathematical algorithms should be employed to search for the sparsest solution. A near optimal solution is achieved by using the l_1 norm based minimization as follows:

$$\min \|\hat{\mathbf{C}}\|_{l_1} \quad \textit{subject to} \quad \mathbf{y}=\mathbf{\Theta C}\,, \tag{13}$$

where $\hat{\mathbf{C}}$ is the Hermite transform vector of reconstructed signal **f**.

In order to solve the previous minimization problem, first we need to calculate the initial Hermite transform using the available set of M_A samples with the time support Ω: $\mathbf{y}=\mathbf{\Phi f}=\mathbf{f}(\Omega)$. Therefore, we observe the signal in the form:

$$f_0(n)=\begin{cases} y(n), & \text{for } n\in\Omega \\ 0, & \text{elsewhere} \end{cases} \tag{14}$$

The initial vector of Hermite transform coefficients can be then calculated using N Hermite functions ($N\leq M$) as follows:

$$\mathbf{C}_0=\mathbf{H}_\Omega\mathbf{y}\ , \tag{15}$$

where $\mathbf{H}_{\Omega}$ contains only the columns of **H** that corresponds to instants $n \in \Omega$. Alternatively, we can write:

$$C_0(i) = \frac{1}{M} \sum_{m=0}^{M-1} \frac{\psi_i(n_m)}{(\psi_{M-1}(n_m))^2} f_0(n) \ , \ i=1,...,N. \tag{16}$$

In order to determine the signal support in the Hermite transform domain, the initial vector of the Hermite transform coefficients $\mathbf{C}_0$ is compared by the threshold T:

$$\mathbf{k} = \arg\{\mathbf{C}_0 > T\}. \tag{17}$$

The exact values of Hermite coefficients at positions selected in vector **k** are obtained as a solution of the CS problem:

$$\Theta_{\mathbf{cs}} \mathbf{C_0} = \mathbf{y}. \tag{18}$$

The CS matrix $\mathbf{\Theta_{cs}}$ is obtained from the inverse Hermite transform matrix $\mathbf{\Theta_{cs}} = \mathbf{\Psi}(\Omega, \mathbf{k})$, using columns that correspond to frequencies **k** and rows corresponding to measurements with support Ω. The system is solved in the least square sense as follows:

$$\mathbf{C} = \left(\mathbf{\Theta}_{cs}^{*} \mathbf{\Theta}_{cs}\right)^{-1} \mathbf{\Theta}_{cs}^{*} \mathbf{y} \tag{19}$$

where (*) denotes the conjugate transpose operation.

Analysis of components reconstruction**:** Let us consider the isometry property of Hermite transform matrix Ψ:

$$\begin{aligned} \|\Psi C\|_2^2 &= \|x\|_2^2 = |x(0)|^2 + ... + |x(N-1)|^2 = \left|\sum_{k=0}^{N-1} C(k)\psi_k(0)\right|^2 + ... + \left|\sum_{k=0}^{N-1} C(k)\psi_k(N-1)\right|^2 = \\ &= \sum_{k=0}^{N-1} C(k)^2 \sum_{n=0}^{N-1} \psi_k(n)^2 + \sum_{k_1=0}^{N-2}\sum_{k_2=0}^{N-1} 2C(k_1)C(k_2) \sum_{n=0}^{N-1} \psi_{k1}(n)\psi_{k2}(n) \end{aligned} \tag{20}$$

The previous equation can be written as follows:

$$\|\Psi C\|_2^2 = \left((C(0))^2 \sum_{n=0}^{N-1} (\psi_0(n))^2 + ... + (C(N-1))^2 \sum_{n=0}^{N-1} (\psi_{N-1}(n))^2 \right) + \sum_{k_1=0}^{N-2}\sum_{k_2=0}^{N-1} 2C(k_1)C(k_2) \sum_{n=0}^{N-1} \psi_{k1}(n)\psi_{k2}(n) \tag{21}$$

Now, observe the first term on the right side given by: $P = C(0)^2 \sum_{n=0}^{N-1} (\psi_0(n))^2 + ... + C(N-1)^2 \sum_{n=0}^{N-1} (\psi_{N-1}(n))^2$, particularly the sum of squared values of different Hermite functions. Fig. 1 illustrates the sum of squared values of different Hermite functions calculated in the zeros of Hermite polynomials for: N=(128, 100, 70 and 50). Unlike in the case of the Fourier transform basis, where the sum of absolute values of complex exponential basis would be constant for any frequency k, from Fig. 1 we can note an approximate low-pass characteristic of the curves, meaning that the lower order coefficients are favored compared to the higher order coefficients. Consequently, when applying the threshold for components detection, it would be easier to detect a set of low order coefficients than the high order ones.

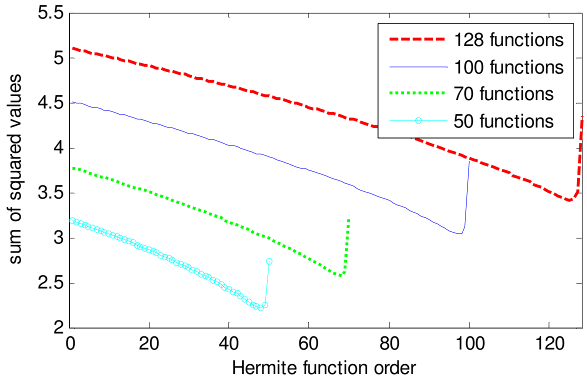


Fig. 1 Sum of squared values of Hermite functions for different *N*

4. Introducing the short-time Hermite transform and short-time combined transform

A) Short-time Hermite transform

Let us assume that N=M in (10) and to define an $M\times M$ Hermite matrix:

$$\mathbf{H}_M = \frac{1}{M}\begin{bmatrix} \frac{\psi_0(0)}{(\psi_{M-1}(0))^2} & \frac{\psi_0(1)}{(\psi_{M-1}(1))^2} & \cdots & \frac{\psi_0(M-1)}{(\psi_{M-1}(M-1))^2} \\ \frac{\psi_1(0)}{(\psi_{M-1}(0))^2} & \frac{\psi_1(1)}{(\psi_{M-1}(1))^2} & \cdots & \frac{\psi_1(M-1)}{(\psi_{M-1}(M-1))^2} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\psi_{M-1}(0)}{(\psi_{M-1}(0))^2} & \frac{\psi_{M-1}(1)}{(\psi_{M-1}(1))^2} & \cdots & \frac{\psi_{M-1}(M-1)}{(\psi_{M-1}(M-1))^2} \end{bmatrix}. \tag{22}$$

The short-time Hermite transform (STHT) can be defined as a composition of Hermite transform matrices whose size is defined by the window width. Without loss of generality, we may assume the non-overlapping unit rectangular windows (with the width of M samples). The total transform matrix for the STHT can be created as follows:

$$\mathbf{W} = \mathbf{I}_{L/M} \otimes \mathbf{H}_M = \begin{bmatrix} \mathbf{H}_M & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_M & \dots & \mathbf{0} \\ \dots & \dots & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{H}_M \end{bmatrix}, \tag{23}$$

where $\mathbf{I}$ is identity matrix of size ($L/M \times L/M$), $\mathbf{0}$ is $M \times M$ zero matrix, L is the total length of the signal, while $\otimes$ denotes the Kronecker product. The STHT can be defined using transform matrix $\mathbf{W}$ as:

$$\mathbf{STHT} = \mathbf{Wf} = \mathbf{W}\Psi_L\mathbf{C}, \tag{24}$$

where $\mathbf{f}$ is a time domain signal vector of size $L \times 1$, $\mathbf{C}$ is a Hermite transform vector of size $L \times 1$, while $\mathbf{STHT}$ is a column vector containing all STHT vectors STHT_{Mi} (m_i), $i = 0,1,..., P$ (P is the number of windows). Furthermore we may write:

$$\mathbf{STHT} = \mathbf{A}\,\mathbf{C}, \tag{25}$$

where $\mathbf{A} = \mathbf{W}\Psi_L$ is matrix of size $L \times L$ that maps the global transform domain information in $\mathbf{C}$ into local information in $\mathbf{STHT}$. In the case of windows with variable length over time (time-varying windows), the smaller matrices within $\mathbf{W}$ will be of different sizes. Particularly, for a set of P rectangular windows of sizes M_1, M_2, … , M_P instead of $\mathbf{H}_M$ we will have: $\mathbf{H}_{M_1}, \mathbf{H}_{M_2}, ..., \mathbf{H}_{M_P}$.

In the context of compressive sensing, the STHT allows us to define two types of CS problem. A common CS problem is defined by assuming that the missing samples appear in the time domain, while the sparsity is exploited in the transform domain. However, the missing samples may also appear in the STHT domain, after applying certain filter forms such as the L-estimate filtering in the presence of strong noisy pulses. The two CS optimization problems are discussed in the sequel.

1) Assume that CS procedure is done in the time domain, such that the measurements are taken from each windowed signal part: $\mathbf{y}_i = \mathbf{f}_{M_i}(\Omega)$. Based on the definitions given in Section III, the minimization problem is given by:

$$\min \|\mathbf{C}_i\|_1 \quad \textit{subject to} \quad \mathbf{y}_i = \Psi_M \mathbf{C}_i, \tag{26}$$

for i=1, 2, …, P, where P is the total number of windows, while Ψ_M is the inverse Hermite transform of size $M \times M$ corresponding to $\mathbf{H}_M$ from $\mathbf{W}$.

2) Let us observe the missing values in the STHT domain, where the measurements vector is denoted by $\mathbf{STHT}_{cs}$. The CS problem formulation is defined using the linear relationship (25). The CS matrix, denoted by $\mathbf{A}_{cs}$, is formed by omitting the rows from $\mathbf{A}$ corresponding to the positions of missing values in $\mathbf{STHT}_{cs}$. Then, a simple linear relationship can be established between the reduced observations in $\mathbf{STHT}_{cs}$ and the sparse Hermite transform vector $\mathbf{C}$ (corresponding to the entire signal). Hence, the CS problem is given in the form:

$$\min \|\mathbf{C}\|_1 \quad \textit{subject to} \quad \mathbf{STHT}_{cs} = \mathbf{A}_{cs}\mathbf{C}. \tag{27}$$

It means that the missing samples in the **STHT** can be reconstructed such as to provide the best concentrated **C**. The above CS optimization problem can be solved using various existing CS reconstruction algorithms.

B) Short-time combined transform

On the basis of the STHT, we can also define the short-time combined transform as follows:

$$\mathbf{X} = \mathbf{Z}\mathbf{f}, \tag{28}$$

where the transform matrix **Z** is made as a combination of Hermite transform matrices and other transforms:

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}_{M_1} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_{M_2} & \dots & \mathbf{0} \\ \dots & \dots & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{Z}_{M_P} \end{bmatrix}, \tag{29}$$

where $\mathbf{Z}_{Mi}$ can be either the $M_i \times M_i$ Hermite transform or some other suitable transform matrix of the same size. For instance, we may observe one example of the combination of Hermite transform and Fourier transform as a special case:

$$\mathbf{Z}=\begin{bmatrix} \mathbf{H}_M & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{F}_M & ... & \mathbf{0} \\ ... & ... & \mathbf{F}_M & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{H}_M \end{bmatrix}. \tag{1.30}$$

The combined matrix $\mathbf{Z}$ is thus composed of Hermite transform matrix $\mathbf{H}$ of size $M \times M$, two sequential Fourier transform matrices $\mathbf{F}$ of size $M \times M$, and again one Hermite transform matrix of the same size. In the case of compressive sensing scenario, the combined

$$\begin{bmatrix} \mathbf{X}_{M_1} \\ \mathbf{X}_{M_2} \\ ... \\ \mathbf{X}_{M_P} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{M_1} & \mathbf{0} & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{Z}_{M_2} & ... & \mathbf{0} \\ ... & ... & ... & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & ... & \mathbf{Z}_{M_P} \end{bmatrix} \begin{bmatrix} \mathbf{f}_{M_1} \\ \mathbf{f}_{M_2} \\ ... \\ \mathbf{f}_{M_P} \end{bmatrix}, \tag{31}$$

where $\mathbf{f}_{M_1}$ represents the vector containing first M_1 samples in $\mathbf{f}$, vector $\mathbf{f}_{M_2}$ contains the next M_2 samples from $\mathbf{f}$, while $\mathbf{f}_{M_P}$ contains the last M_P samples from $\mathbf{f}$ such that $M_1+M_2+...+M_P=L$.

We can again observe the set of measurements corresponding to different signal parts: $\mathbf{y}_i=\mathbf{f}_{M_i}(\Omega)$ where $\mathbf{\Theta}_{M_i}=inv\left(\mathbf{Z}_{M_i}\right)$. Therefore, the CS minimization problem is given by:

$$\min \|\mathbf{X}_{M_i}\|_1 \quad \textit{subject to} \quad \mathbf{y}_i=\mathbf{\Theta}_{M_i}\mathbf{X}_{M_i}, \tag{32}$$

for i=1, 2, …, P, where the inverse transform denoted by $\mathbf{\Theta}_{M_i}$ can be changed in each window depending on the signal characteristics.

5. EXPERIMENTAL EVALUATION

Example 1: In order to illustrate efficiency of the proposed method, let us observe a signal that is sparse in the Hermite transform domain. The time domain signal is shown in Fig. 2a. The Hermite transform of the observed signal consists of ten components with unit amplitudes, as shown in Fig. 2b. The available number of samples is

60% of the total signal length, while the maximal order of Hermite functions used in the expansion is limited to N=60.

After calculating the initial Hermite transform using available signal samples, the threshold is applied to select signal support in the Hermite domain. The threshold is set empirically to T=αmax{$\mathbf{C}$}, α=0.4, after performing a large number of experiments. Moreover, the results are not very sensitive on threshold settings, and even lower thresholds (e.g. α=0.3, α=0.2) can be used, because all additional (false) components selected by such a threshold would obtain their true zero-values afterwards. The suitable threshold can be even theoretically derived (under certain assumptions). Namely, the effects caused by missing samples in time domain can be modeled by the noise in the Hermite transform domain. The noise can be described using two random variables: the first one corresponds to the noise appearing at non-signal components (noise-alone), while the second one corresponds to the noise appearing at the signal components positions. The two random variables can be described by the corresponding mean values and variances that could be further employed to define the threshold level. Namely, with certain probability, we may define the threshold value that is just above the level of noise components. Consequently, such a threshold would select only the signal components determining the signal support used in the reconstruction procedure. Since the threshold derivation would require significant space and analyzes, it could be a topic of some further work.

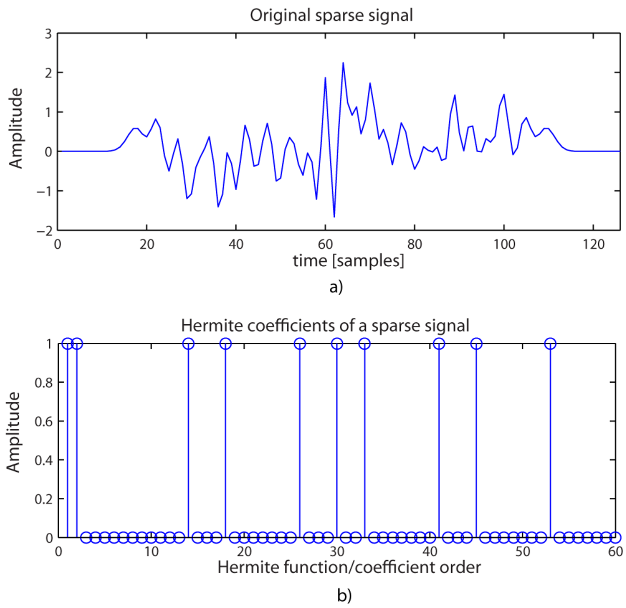


Fig. 2 The original signal: *a*) Time domain representation, b) Hermite transform of signal calculated using *N*=60 Hermite functions

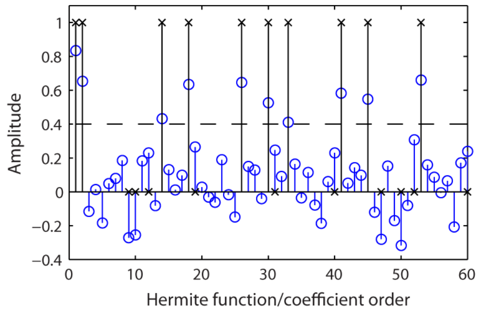


Fig. 3 Hermite coefficients of signal with missing samples –o, and the Reconstructed Hermite coefficients –x

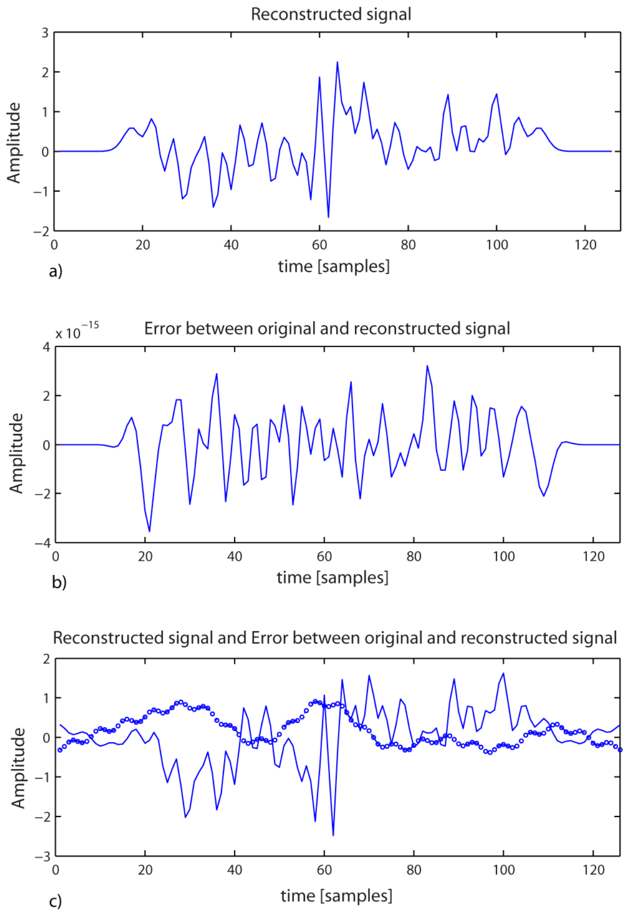


Fig. 4 The reconstructed signal: a) Time domain representation, b) The absolute error between original and reconstructed signal, c) For comparison: reconstructed signal based on the Fourier transform reconstruction approach(-o) and the corresponding error (solid line)

The selected components in the Hermite transform domain are shown in Fig. 3. Using the detected positions of components, the exact values of Hermite coefficients are calculated according to (19), (Fig. 3, marked by -x). The reconstructed signal, as well as the absolute error between original and reconstructed signal are shown in Fig. 4 (*a* and *b*, respectively). For the comparison, the reconstruction based on the Fourier transform is considered. However, it is shown that the Fourier transform based reconstruction cannot be used in this case, because it produces a significant error (Fig. 4c, 10 largest Fourier transform components are used for reconstruction). The reason is in the fact that the signals that are sparse in Hermite transform domain, do not exhibit sparsity in the Fourier transform domain (or some other transform domains). This means that the exact results, in this case, can be obtained only using the Hermite transform providing the sparse signal representation. Similar conclusion holds for other transform domains where the signal is not sparse (Fractional Fourier, Wavelet, etc.).

Example 2: In this example we have observed different sparse signals (having 10 components) in the Hermite domain through the 1000 repetitions of the previously described procedure. The aim is to test how the accuracy of the proposed method changes for different number of missing samples. Namely, the number of missing samples was increased in steps of 10% starting from zero up to 80% of missing samples. For each instance on the y-axes, we performed 1000 iterations with different 10-component signals, and the mean square errors between original and reconstructed signals are calculated and shown in Fig. 5a. We can observe that up to 70% of missing samples, the MSE is still low, but it starts to increase significantly afterwards (for values higher than 70%). This means that the reliable reconstruction cannot be guaranteed for such a large number of missing samples (such as 80% of missing samples).

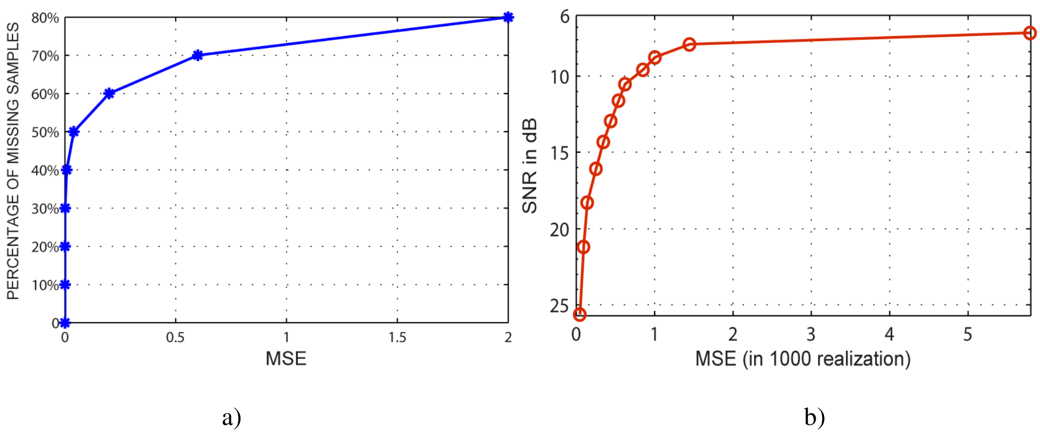


Fig. 5 a) MSE calculated for 1000 realization and for different number of missing samples, b) MSE calculated for different values of SNR in 1000 realizations of external noise

Additionally, the proposed approach is tested in the presence of external additive Gaussian noise. Namely, the MSE is calculated for different values of SNR (for each SNR, we assume 1000 realizations of random noise). The results are presented in Fig.5b, showing that the proposed approach can be efficiently used for SNR>8dB. For SNR<8 dB the MSE starts to increase rapidly.

Example 3: This example illustrates the concept of combined Hermite-Fourier transform and CS reconstruction. Observe the signal with the total length of L=32 samples composed of two parts: first part exhibit sparsity in the Hermite transform domain with K=2 components (HT components) at positions 7 and 12 with amplitudes 0.8 and 0.65; second part exhibit sparsity in the Fourier transform domain with K=2 components (FT components) at the positions -3 and 4 with amplitudes 1 and 0.8, respectively. Now consider the compressive sensing scenario with only 50% of samples available (original full length signal and available measurements are shown in Fig. 6). The original short-time combined transform obtained using windows width equal to 16 samples is shown in Fig. 7a, where the white fields represent 0 value. Thus, the signal represented by a full set of samples is sparse meaning that it is fully concentrated on a few non-zero components. In the case when we deal with 50% of missing samples, the corresponding short-time combined transform is shown in Fig. 7b. Note that in this case the sparsity is disturbed as a consequence of missing samples. Instead of zero values at non-signal position, the noise appear as a consequence of missing samples.

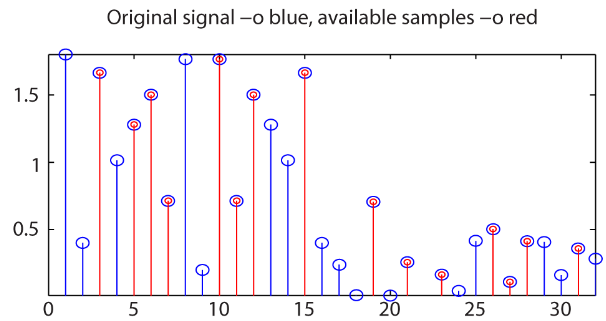
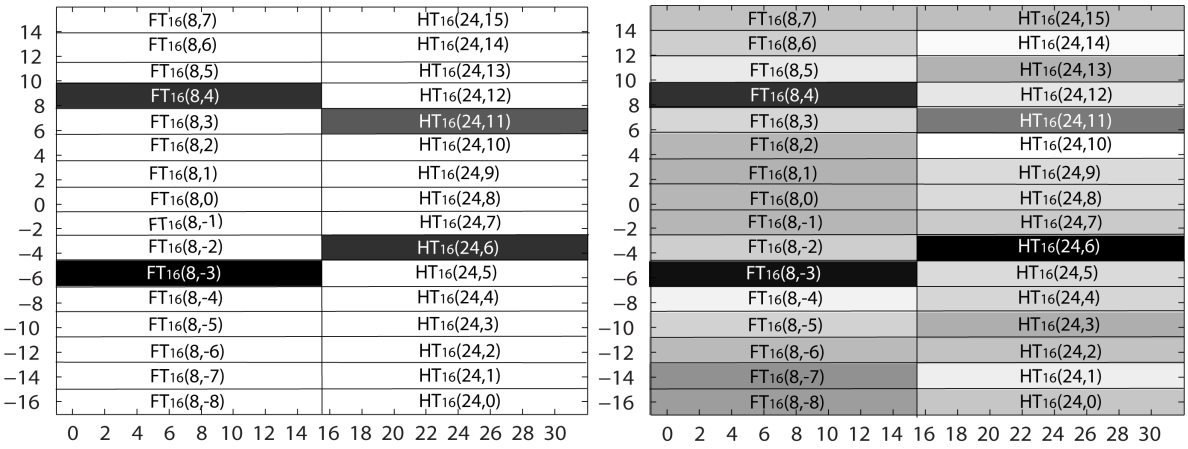
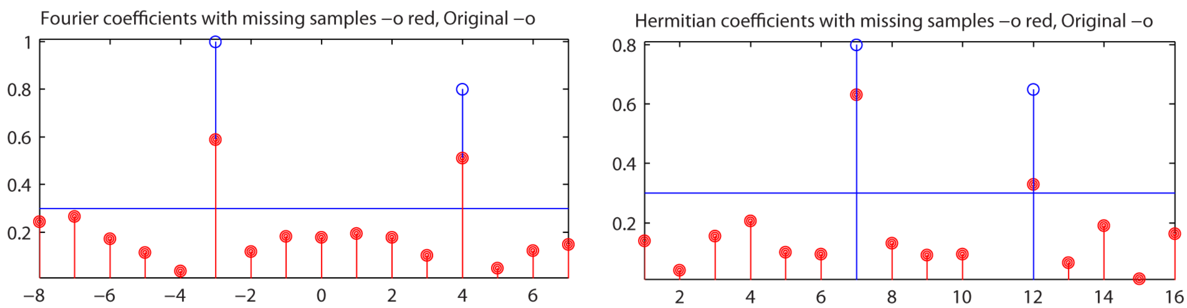


Fig. 6 Original full length signal and available measurements in time domain



a) **b)**

Fig. 7 a) Original short-time combined transform of full length signal (FT-Fourier transform, HT- Hermite transform), b) short-time combined transform of available measurements



a) **b)**

Fig. 8. Selecting the components of interest by applying threshold to: a) FT components, b) HT components

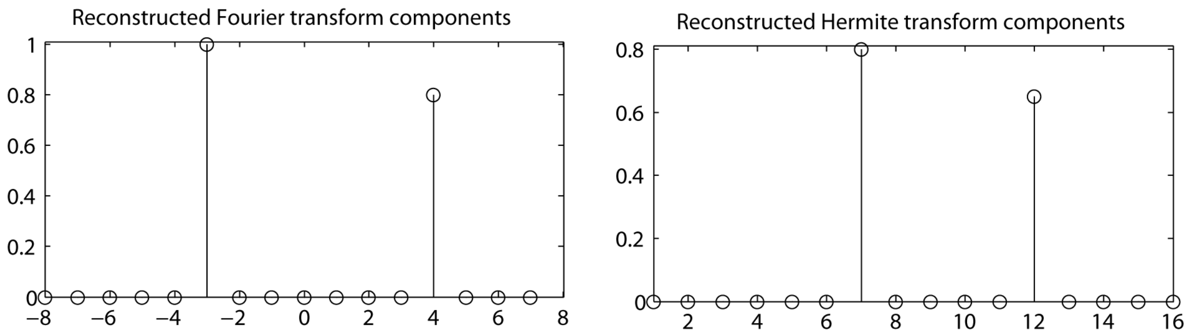


Fig. 9. Reconstructed signal components

The threshold based components selection and signal reconstruction (in analogy to (17) and (19)) is applied to both parts of the signal, FT components and HT components (Fig. 8a and b, respectively). Based on the identified positions of components, the exact signal reconstruction is done using the procedure presented in Section 3. The reconstructed signal components are shown in Fig. 9.

6. Conclusion

The possibility of using the Hermite transform in Compressive sensing applications was explored in this work. The Compressive sensing set up and signal reconstruction approach in the Hermite transform domain was defined. A simple and fast procedure for the total reconstruction of signals using selected Hermite transform coefficients, provides the results very close to the original signal even when we deal with significant missing information. It is important to emphasize that the proposed concept in the Hermite transform domain can be also combined with other known Compressive sensing solvers. The entire concept is generalized and extended by defining the short-time Hermite transform as well as the short-time combined transform. These two transforms open more possibilities to apply the compressive sensing approach in different scenarios.

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