Reconstruction and classification of wireless signals based on Compressive Sensing approach

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Abstract—**The procedure for the classification and reconstruction of randomly under-sampled signals transmitted through the communication channel, is proposed in this paper. The focus of this work is on the wireless communication signals that operate in the same frequency band and may interfere with each other. In the first stage, the separation of signal components is done by applying the concept of eigenvalue decomposition. Next, the compressive sensing approach is used to reduce the number of transmitted samples and to provide accurate signal reconstruction upon transmission. In the last step, the classification is done by observing the time-frequency characteristics of reconstructed separated components. The theory is proved by the experimental results.**

Keywords-component; Compressive Sensing, Eigenvalue Decomposition, FHSS signal, IEEE 802.11b, reconstruction, signal separation, under-sampling

I. INTRODUCTION

Wireless technology uses radio waves for the transmission of the useful information. Systems using the radio waves should satisfy certain requirements, such as low power consumption and high speed transmission. Therefore, depending on the system requirements, various standards in wireless communications are developed, such as: WLAN, WWAN, WPAN, WWAN, etc. [1]-[3]. The standards differ in the data rates, energy consumption, signal modulations, operation distances, etc. Since some of the standards use the same operational frequency band, the interferences may appear between signals belonging to different standards.

Our focus in this paper is on the signals belonging to the Bluetooth (Frequency Hopping Spread Spectrum-FHSS signals) and IEEE 802.11b standards, both operating in the Industrial, Scientific and Medical (ISM) frequency band [1]. Both standards deal with the sinusoidal signals, but with different physical characteristics. Namely FHSS signal has short duration components, while the components of the IEEE 802.11b signal have longer time duration. These features of the signal components will be exploited for the classification of the signal components, after decomposition which will be done by using the Eigenvalue Decomposition method (EVD) [5], [6].

EVD has numerous practical applications. It is mainly used for characterization of signals and their components. Here, we apply the EVD to separate components of the initial multicomponent signal. EVD of the original wireless signal, consisted of different components, will produce eigenvectors that correspond to the separated components. Further, in order to increase the transmission efficiency and to decrease the amount of transmitted data, only a small percent of samples per component is considered. This means that the eigenvectors are randomly under-sampled to produce reduced set of data. At the receiver side, all signal components should be completely reconstructed from this small set of samples.

For the reconstruction of randomly under-sampled eigenvectors at the receiver side, the Compressive Sensing (CS) approach is employed [7]-[13]. The CS allows high accuracy reconstruction of signals from an incomplete dataset called measurements, by using optimization algorithms [9]-[12]. It is important to note that the measurement procedure should satisfy certain conditions. The first condition is that signal needs to be sparse in certain domain, and this condition is satisfied for both considered wireless signal types. The second condition is incoherence which is achieved by random selection of measurements (samples) from the observed eigenvectors. After the reconstruction, different components are classified based on their characteristics in the time-frequency domain. A suitable time-frequency representation is obtained using the S-method and classification is done based on the time duration of signal components.

The paper is structured as follows: Section II provides theoretical background on the approaches used for decomposition and reconstruction of under-sampled components. Section III describes the procedure for transmission and classification of the signals, while Section IV provides experimental results on synthetic signals. Conclusion is given in Section V.

II. THEORETICAL BACKGROUND

A. EigenValue Decomposition

In order to efficiently analyze and classify multicomponent signals, one of the possible approaches is to separate signal components and to observe each component separately. In that sense, we consider the eigenvalue decomposition method [4]-

[6]. The EVD of the properly chosen matrix results in eigenvalues and eigenvectors. The eigenvectors correspond to the signals components, while the eigenvalues correspond to their energy. The eigenvalue decomposition method can be defined in time-frequency domain, by using the S-method [4]. In that sense, the autocorrelation matrix can be defined as:

$$\mathbf{A_C} = x(n)x^*(n), \tag{1}$$

where $x(n)$ is a monocomponent signal, and $x^*(n)$ are complex conjugate values of the vector $x(n)$. For the signal with M components, the autocorrelation matrix will be:

$$\mathbf{A_C^M} = \sum_{i=1}^{M} x_i(n)x_i^*(n). \tag{2}$$

The right side of the relation (2) could be defined by using inverse form of the Wigner distribution:

$$\sum_{i=1}^{M} x_i(n+m)x_i^*(n-m) = = \frac{1}{N+1}\sum_{k=-N/2}^{N/2}\sum_{i=1}^{M} WD_i(n,k)e^{j\frac{2\pi}{N+1}2mk}. \tag{3}$$

The Wigner distribution of signal components (if there is no overlapping in the TF plane) is equal to the S-method of the multicomponent signal. Therefore, (3) can be modified as follows:

$$\sum_{i=1}^{M} x_i(n+m)x_i^*(n-m) = \sum_{k=-N/2}^{N/2} SM(n,k)e^{j\frac{4\pi}{N+1}mk}, \tag{4}$$

In other words,

$$A_C^M(n+m,n-m) = \frac{1}{N+1}\sum_{k=-N/2}^{N/2} SM(n,k)e^{j\frac{4\pi}{N+1}mk}. \tag{5}$$

where $\mathbf{A_C^M}$ is a square autocorrelation matrix. Then the eigenvalue decomposition of the square matrix $\mathbf{A_C^M}$ could be written as:

$$\mathbf{A_C^M} = \sum_{i=1}^{N+1} \lambda_i u_i(n)u^*_i(n), \tag{6}$$

where λ_i are eigenvalues and u_i are eigenvectors of matrix $\mathbf{A_C^M}$, such that the eigenvectors correspond to the separated signals components.

B. *Compressive Sensing*

Compressive Sensing (CS), as a new approach to the signal acquisition, provides successful reconstruction of the signal from the small number of available samples (measurements). Random distribution of measurements is required in order to successfully reconstruct the signal. Also, the sparsity of the signal is necessary condition to be met in order to apply the CS approach. Large number of signals in real applications exhibit the sparsity property in certain transformation domain. The sparsity means that there exists a domain in which majority of the coefficients are zero valued and information about the signal is condensed in the small number of non-zero coefficients. If the discrete signal x of length N is sparse in the transform domain Ψ, the signal can be represented in terms of basis matrix as follows:

$$x = \sum_{i=1}^{N} \mathbf{X}_i\psi_i = \Psi\mathbf{X}, \tag{7}$$

where $\mathbf{X}$ denotes vector of transform domain coefficients (where only $K<<N$ coefficients are non-zero). The vector of available measurements $\mathbf{y}$ (of length $M<N$) is formed according to:

$$\mathbf{y} = \Theta\Psi\mathbf{X}. \tag{8}$$

Matrix Θ is used to model random selections of the original signal samples. The system of equations (8) is undetermined since $M<N$ and in order to obtain unique solution, the complex mathematical optimization algorithms are used [9]-[13].

III. A Procedure for Decomposition, CS Reconstruction and Classification of Wireless Signals

In the sequel, the procedure for separation of signal components, their under-sampling, transmission and reconstruction from a reduced set of samples, is described. The procedure is illustrated using the diagram in Figure 1. The goal of the procedure is to separate the interfering standards: FHSS and IEEE 802.11b, and, at the same time, to enable transmission at lower rates.

1) First, the short-time Fourier transform (STFT) and TF representation of the input signal are calculated. The STFT of the discrete signal $x(n)$, with the sliding window $w(n)$ is defined as follows:

$$STFT(n,k) = \sum_{m=-N/2}^{N/2-1} w(m)x(n+m)e^{-j2\pi mk/N}, \tag{9}$$

Based on the STFT, we calculate the S-method as a quadratic TF distribution, that provides good concentration of the auto components of the signal, and, at the same time, avoids cross terms in the observed multicomponent signals. The S-method is defined as:

$$SM(n,k) = \sum_{i=-L}^{L} P(i)STFT(n,k+i)STFT^*(n,k-i), \tag{10}$$

where $P(i)$ denotes window function in the frequency domain.

2) Next, the autocorrelation matrix $\mathbf{A_C}$ is calculated as inverse of the S-method. The EVD is applied on the matrix $\mathbf{A_C}$ resulting in eigenvalues and eigenvectors.

3) The eigenvectors, corresponding to signal components are under-sampled by randomly choosing certain percent of existing samples. At the receiver side, the vectors are reconstructed by using optimization algorithms. Here, the ℓ_1-norm minimization is used for the vector reconstruction. The ℓ_1-norm minimization is defined as:

$$\mathbf{X} = \min \|\mathbf{X}\|_{\ell_1} \text{ subject to } \mathbf{y} = \Theta\Psi\mathbf{X}, \tag{11}$$

where $\mathbf{X}$ is a solution of the minimization problem. The ℓ_1-norm of the vector $\mathbf{X}$ is defined as:

$$\|\mathbf{X}\|_{\ell_1} = \sum_{i=1}^{N} |\mathrm{X}_i|, \tag{12}$$

Here, the solution of the ℓ_1 minimization problem is based on the basis pursuit primal-dual, using the L1-magic solver.

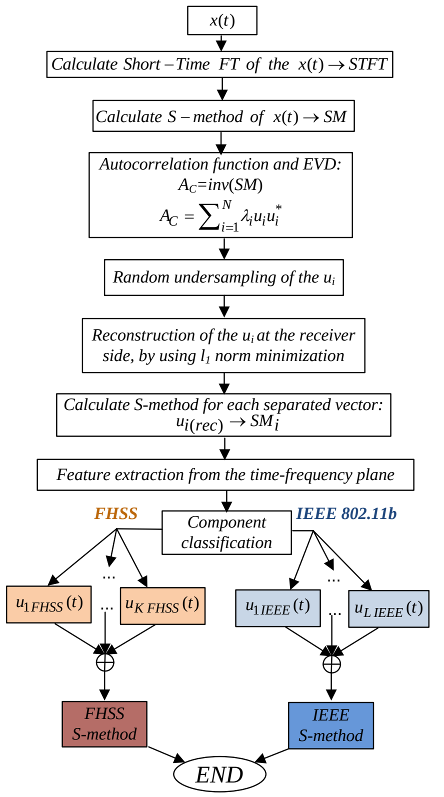

Figure 1. The algorithm for decomposition and classification of the signal components

4) After the components reconstruction, the S-method is calculated - SM_i for each component separately. Then the components features are extracted from the TF plane. The features could be extracted by using TF classification modes, introduced in [14],[15]. Based on these features, the decision is made – component belong either to the IEEE 802.11b signal or to the FHSS standard.

Complexity of the proposed system could be observed by several parts: calculation of the time-frequency representation, eigenvalue decomposition and reconstruction based on the l_1-minimization. STFT calculation in T time instants, by using the N_F samples FFT, requires $O(TN_F log_2 N_F)$ arithmetic operations and S-method calculation adds $TN_F L$ operations. EVD of the quadratic, $N \times N$ matrix, has a complexity of $O(N^3)$ while l_1-minimization (solved by using primal-dual interior point method) has iteration complexity $O(N^3)$ and requires $O(\sqrt{N})$ iterations.

IV. Experimental Results

The procedure is tested by using the synthetic signal, consisted of the two signals from the interfering standards: FHSS and IEEE 802.11b signal. FHSS signal is consisted of four components while IEEE 802.11b is consisted of two components. The S-method of the starting signal is shown in Figure 2. The EVD procedure is applied in order to obtain the eigenvectors. As it can be seen from the S-method of the signal, FHSS signal components are of higher energy compared to the IEEE 802.11b signal components. Therefore, first four eigenvectors obtained from the EVD correspond to the components of the FHSS signal, while fifth and sixth eigenvector correspond to the IEEE 802.11b signal components.

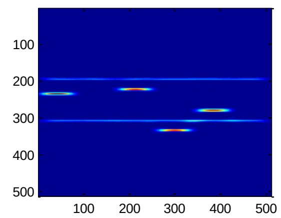

Figure 2. The S-method of the original signal

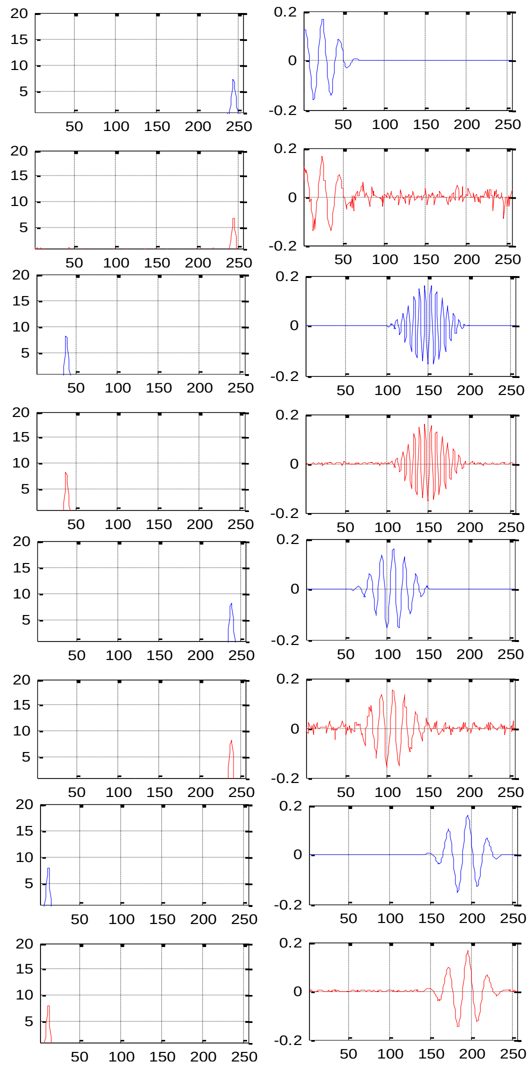

Figure 3. Separated components of the FHSS signal – blue is the original component, red is the CS reconstructed component. Left column is signal DFT, right column is time domain of the signal

Eigenvectors are further randomly under-sampled using only 40% of the total number of samples from each eigenvector. At the receiver side, the eigenvectors are reconstructed from this relatively small set of available samples. The reconstruction is done by using the ℓ_1 minimization. The original (blue) and reconstructed (red) eigenvectors are shown in Figures 3 and 4. Figure 3 shows FHSS signal components, while IEEE 802.11b signal components are shown in Figure 4. Note that the left column in the Figures 3 and 4 corresponds to the component DFT, while the right column represents time domain of the separated components eigenvectors. After the separation and reconstruction of the signal components, the FHSS and IEEE 802.11b signals are formed from the corresponding eigenvectors. Note that now, these two standards are separated. The S-methods of the separated signals are calculated and are shown in Figure 5.

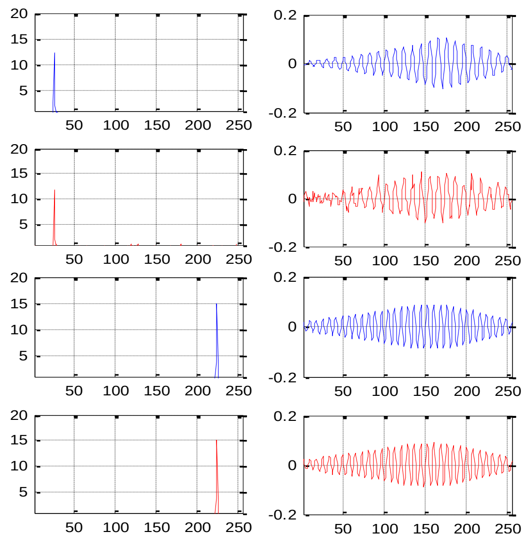

Figure 4. Separated components of the IEEE 802.11b signal – blue is the original component, red is the CS reconstructed component. Left column is signal DFT, right column is time domain of the signal

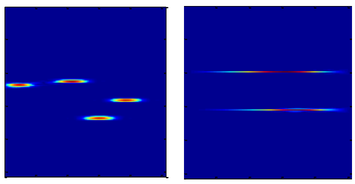

Figure 5. a) S-method of the FHSS signal; b) S-method of the IEEE 802.11b signal

V. CONCLUSION

The procedure for the separation and reconstruction of the interfering wireless signals, is presented in the paper. The separation procedure is based on the EVD, while for the reconstruction the optimization algorithm developed in the CS theory is used. After being separated, components are randomly under-sampled and sent through the communication channel. At the receiver side, components are recovered by using the optimization algorithm based on ℓ_1 -norm minimization. Only 40% of the total number of samples per component is used. After the reconstruction, the S-method of the each component is calculated and components features are estimated. Based on the estimated features, signal can be successfully classified either as FHSS or IEEE 802.11b signal.

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