A Higher Order Time-Frequency Representation With Reduced Cross-Terms

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***Abstract* — A higher order time-frequency (TF) representation, fully concentrated up to the fourth order polynomial phase of the signal is revisited. This representation is based on the first derivative approximation of the phase. We propose a realization based on the concept of windowed frequency convolutions in order to reduce the cross-terms in multi-component signals, following the ideas from the S-method framework. Several numerical examples illustrate and verify the presented theory.**

***Keywords* — Digital signal processing, Non-stationary signals, Time-frequency signal analysis**

I. Introduction

During the last few decades time-frequency (TF) signal analysis has been an emerging research field, followed by numerous applications [1]– [8]. Instantaneous frequency (IF) estimation is one of the most important problems analyzed in wide TF analysis literature [1], [5], [6].

The aim to reveal and extract high level signal characteristics such as the IF has resulted in numerous proposed time-frequency representations (TFR), [1]. Many of these representations provide improvements in useful signal content (auto-terms) concentration at or around the IF, comparing with the frequently used Short-time Fourier transform (STFT) and Wigner distribution (WD) [3], [4]. Main problems regarding higher-order representations include high numerical complexity, noise sensitivity, demanding parameters search, and the appearance of undesirable cross-terms in multi-component signals [1], [2].

Following the idea to define a representation based on signal's phase first derivative approximation, a higher order representation being able to ideally concentrate signals having up to the fourth order polynomial phase is proposed in [7]. However, in the case of multi-component signals, this representation produces strong cross-terms, preventing a successful IF estimation. In this paper we present an algorithm for the realization of this TFR with significantly reduced cross-terms.

The paper is organized as follows. In Section II basic theory regarding the analyzed higher order TFR is presented. Its reduced cross-terms realization is introduced in Section III. Numerical results illustrating and confirming the theory are presented in Section IV, whereas the paper ends with concluding remarks.

II. Background theory

Starting from the analytic signal definition

$$x(t) = A(t)e^{j\phi(t)}, \tag{1}$$

and assuming slow amplitude variations comparing with the instantaneous phase variations $|A'(t)| \ll |\phi'(t)|$, the IF is defined as

$$\Omega(t) = \frac{d\phi(t)}{dt}. \tag{2}$$

An ideal TFR can be introduced in the following form:

$$ITF(t,\Omega) = 2\pi|A(t)|^2\delta(\Omega - \phi'(t)), \tag{3}$$

fully concentrating signal energy at the IF. One out of many open topics in the TF signal analysis deals with the development of representations having the form and properties as close as possible to the ITF. This TFR can be considered as the Fourier transform (FT) of function

$$R(t,\tau) = |A(t)|^2 e^{j\phi'(t)\tau}. \tag{4}$$

As the signal phase first derivative can be approximated with [1]:

$$\Omega(t) \approx \frac{\sum_i b_i\phi(t+c_i\tau)}{\tau} = \frac{d\phi(t)}{dt} + O(\phi^{(p)}(\tau)), \tag{5}$$

the general form of function (4) whose FT produces ITF follows

$$R(t,\tau) = \prod_i x^{b_i}(t+c_i\tau). \tag{6}$$

Under the assumption of slow amplitude variations, its influence is neglected in further analysis. General form of TFR based on (6) reads

$$GD(t,\Omega) = \int_{-\infty}^{\infty} \prod_i x^{b_i}(t+c_i\tau)e^{-j\Omega\tau}d\tau. \tag{7}$$

Expanding term $b_i\phi(t+c_i\tau)$ in Taylor series around t

$$b_i\phi(t+c_i\tau) \approx b_i\phi(t) + b_i\phi^{'}(t)c_i\tau + b_i\phi^{''}(t)\frac{(c_i\tau)^2}{2!} + \\ + b_i\phi^{'''}(t)\frac{(c_i\tau)^3}{3!} + b_i\phi^{(4)}(t)\frac{(c_i\tau)^4}{4!} + ...$$

coefficients b_i i c_i follow with the conditions that [1]:

- The sum of coefficients with $\phi(t)$ is equal to 1, eliminating the signal phase influence;
- The sum of coefficients with $\phi^{'}(t)$ is equal to 1, as the goal is the first derivative approximation;
- The sum of coefficients with $\phi^{(n)}(t)$ is equal to 0 up to the desired order.

Either following these conditions, or directly using the well-known first derivative approximation of the form

$$\Omega(t) \approx \frac{\phi(t-\frac{\tau}{6}) - 8\phi(t-\frac{\tau}{12}) + 8\phi(t+\frac{\tau}{12}) - \phi(t+\frac{\tau}{6})}{\tau} \\ = \frac{d\phi(t)}{dt} + O(\phi^{(5)}(\tau)),$$

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according to (6) we get the function

$$R(t,\tau) = x(t-\frac{\tau}{6})x^{*8}(t-\frac{\tau}{12})x^{8}(t+\frac{\tau}{12})x^{*}(t+\frac{\tau}{6}), \quad (8)$$

whose FT is a new TF representation fully concentrating signals up to the fourth order polynomial phase. Its form reads

$$PD(t,\Omega) = \int_{-\infty}^{\infty} x(t-\frac{\tau}{6})x^{*8}(t-\frac{\tau}{12})\times \times x^{8}(t+\frac{\tau}{12})x^{*}(t+\frac{\tau}{6})e^{-j\Omega\tau}d\tau. \quad (9)$$

The discretization over the time and lag with $t = m\Delta t$ and $\tau = n\Delta t$, with sampling period $\Delta t = 1/(12 f_{\max})$, and the discretization of frequency $\omega = \Omega\Delta t$ using $\omega = \pi k/6N$, lead to the discrete form of this representation

$$PD(n,k) = 12\sum_{m=-6N}^{6N-1} x(n-2m)x^{*8}(n-m)\times \times x^{8}(n+m)x^{*}(n+2m)e^{-\frac{j2\pi mk}{N}} \quad (10)$$

suitable for numerical implementations. Maximal frequency in signal spectrum is denoted with $f_{\max}$. Note that 6 times more samples are needed for aliasing-free calculation of this representation, compared with the corresponding pseudo-WD with the same window duration.

III. Proposed realization

A. Continuous windowed frequency convolutions

Let us observe the definition (9) of higher order representation $PD(t,\Omega)$, assuming, for simplicity, unit symmetric lag window $w(\tau) = 1$, $-T/2 \le \tau \le T/2$. For each observed time instant t it can be understood as a frequency domain convolution of the form

$$PD(t,\Omega) = R_{x^2}(t,\Omega) *_{\Omega} R_{x^{16}}(t,\Omega), \quad (11)$$

with the following definitions

$$R_{x^2}(t,\Omega) = FT\left[x(t-\frac{\tau}{6})x^{*}(t+\frac{\tau}{6})\right], \quad (12)$$

$$R_{x^{16}}(t,\Omega) = FT\left[x^{*8}(t-\frac{\tau}{12})x^{8}(t+\frac{\tau}{12})\right]. \quad (13)$$

Operator $FT[\cdot]$ denotes the Fourier transform over variable τ. Furthermore, introducing notation $S_{n_1}(t,\Omega) = FT\left[x(t-\frac{\tau}{6})\right]$ and $S_{p_1}(t,\Omega) = FT\left[x^{*}(t+\frac{\tau}{6})\right]$ equation (12) can be also represented in form of a frequency domain convolution

$$R_{x^2}(t,\Omega) = S_{n_1}(t,\Omega) *_{\Omega} S_{p_1}(t,\Omega). \quad (14)$$

Following the same approach, introducing

$$W_{x^2}(t,\Omega) = S_{n_2}(t,\Omega) *_{\Omega} S_{p_2}(t,\Omega), \quad (15)$$

with $S_{n_2}(t,\Omega)$ and $S_{p_2}(t,\Omega)$ being defined as $S_{n_2}(t,\Omega) = FT\left[x^{*}(t-\frac{\tau}{12})\right]$ and $S_{p_2}(t,\Omega) = FT\left[x(t+\frac{\tau}{12})\right]$, we rewrite (13) in terms of frequency convolutions

$$R_{x^{16}}(t,\Omega) = W_{x^2}(t,\Omega) *_{\Omega} W_{x^2}(t,\Omega) *_{\Omega} *_{\Omega} W_{x^2}(t,\Omega) *_{\Omega} W_{x^2}(t,\Omega). \quad (16)$$

It can be assumed that components $S_{p_2}(t,\Omega)$ are localized in frequency, such that $S_{p_2}(t,\Omega)$ centered at any Ω_0 is spread over a region $[\Omega_0 - \Omega_L/2, \Omega_0 + \Omega_L/2]$. This means that values of $S_{p_2}(t,\Omega)$ appart from Ω_0, that is, outside this region, are not related with value of $S_{p_2}(t,\Omega_0)$, for observed instant t. It should be noted that there is no assumption regarding the exact location of central frequency Ω_0. In the multi-component signal case, each component is localized in its own region.

As the symmetric lag window is assumed, the analyzed component is localized within same frequency region $[\Omega_0 - \Omega_L/2, \Omega_0 + \Omega_L/2]$ in $S_{n_2}(t,\Omega) = FT\{x^{*}(t-\frac{\tau}{6})\}$, also around central frequency Ω_0. Please note that the change in the lag sign is compensated by the signal conjugation. Hence, (15) can be further written as:

$$W_{x^2}(t,\Omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{p_2}(t,\varpi)S_{n_2}(t,\Omega-\varpi)d\varpi = \quad (17)$$

$$\frac{1}{4\pi}\int_{-\Omega_L}^{\Omega_L} S_{p_2}(t,\frac{\Omega}{2}+\frac{\varpi_1}{2})S_{n_2}(t,\frac{\Omega}{2}-\frac{\varpi_1}{2})d\varpi_1,$$

where substitution $\varpi_1 = \Omega/2 - \varpi/2$ is exploited. The integral limits in (17) are changed according to the localization assumption and doubled having in mind the length of the resulting convolution. The signal component in (17) is therefore spread over region $[2\Omega_0 - \Omega_L, 2\Omega_0 + \Omega_L]$. Note that this analysis correspond to the well-known S-method [3] principles, leading to a significant cross-terms reduction. The aim is to improve the signal concentration further applying this principle, in order to obtain the higher order representation $PD(t,\Omega)$. The next term, $R_{x^4}(t,\Omega)$ can be calculated as

$$R_{x^4}(t,\Omega) = W_x(t,\Omega) *_{\Omega} W_x(t,\Omega) = = \frac{1}{4\pi}\int_{-2\Omega_L}^{2\Omega_L} W_x(t,\frac{\Omega}{2}+\frac{\varpi}{2})W_x(t,\frac{\Omega}{2}-\frac{\varpi}{2})d\varpi,$$

with new frequency region where the component is spread being $[4\Omega_0 - 2\Omega_L, 4\Omega_0 + 2\Omega_L]$. The same approach is applied until the highest order term is obtained

$$R_{x^{16}}(t,\Omega) = R_{x^8}(t,\Omega) *_{\Omega} R_{x^8}(t,\Omega) = \quad (18)$$

$$= \frac{1}{4\pi}\int_{-8\Omega_L}^{8\Omega_L} R_{x^8}(t,\frac{\Omega}{2}+\frac{\varpi}{2})R_{x^8}(t,\frac{\Omega}{2}-\frac{\varpi}{2})d\varpi$$

having region of interest $[16\Omega_0 - 8\Omega_L, 16\Omega_0 + 8\Omega_L]$.

In order to calculate (14) for the same analyzed component, frequency regions where the component is spread in $S_{n_1}(t,\Omega)$ and $S_{p_1}(t,\Omega)$ have to be related with corresponding region $[\Omega_0 - \Omega_L/2, \Omega_0 + \Omega_L/2]$ for terms $S_{p_2}(t,\Omega)$ and $S_{n_2}(t,\Omega)$. Based on the definitions of $S_{n_1}(t,\Omega)$ and $S_{p_1}(t,\Omega)$ it can be concluded that the components are spread over the same frequency region in these two terms. Let us relate the region of the component in term $S_{p_1}(t,\Omega) = FT\left[x^{*}(t+\frac{\tau}{6})\right]$ with the frequency region $[\Omega_0 - \Omega_L/2, \Omega_0 + \Omega_L/2]$ where the component is spread in term $S_{p_2}(t,\Omega) = FT\left[x(t+\frac{\tau}{12})\right]$. Conjugate operator appearing in definition of $S_{p_1}(t,\Omega)$ causes the opposite direction of frequency axis compared with $S_{p_2}(t,\Omega)$. Having in mind the lags $\frac{\tau}{12}$ and $\frac{\tau}{6}$ ratio, the component appearing in $S_{p_2}(t,\Omega)$ at central frequency Ω_0 appears at frequency $-2\Omega_0$ in term $S_{p_1}(t,\Omega)$, having a twice larger bandwidth than the component in $S_{p_2}(t,\Omega)$. Consequently, the resulting frequency region for $S_{n_1}(t,\Omega)$ and $S_{p_1}(t,\Omega)$ is $[-2\Omega_0-\Omega_L, -2\Omega_0+\Omega_L]$.

The expression (14) now can be calculated as

$$R_{x^2}(t,\Omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}S_{p_1}(t,\varpi)S_{n_1}(t,\Omega-\varpi)d\varpi \tag{19}$$
$$=\frac{1}{4\pi}\int_{-2\Omega_L}^{2\Omega_L}S_{p_1}(t,\frac{\Omega}{2}+\frac{\varpi}{2})S_{n_1}(t,\frac{\Omega}{2}-\frac{\varpi}{2})d\varpi.$$

The frequency region of interest for $R_{x^2}(t,\Omega)$ becomes $[-4\Omega_0-2\Omega_L, -4\Omega_0+2\Omega_L]$.

Combining (18) and (19) with (11) the resulting cross-terms free representation can be calculated as follows

$$PD(t,\Omega)=R_{x^2}(t,\Omega)*_\Omega R_{x^{16}}(t,\Omega) \tag{20}$$
$$=\frac{1}{4\pi}\int_{-8\Omega_L}^{8\Omega_L}R_{x^2}(t,-\frac{\Omega}{4}-\frac{\varpi}{4})R_{x^{16}}(t,\Omega-\varpi)d\varpi.$$

B. Numerical implementation

Let us observe, for a fixed instant t the samples corresponding to $x(t-\frac{\tau}{6})$, $x^*(t+\frac{\tau}{6})$, $x^*(t-\frac{\tau}{12})$ and $x(t+\frac{\tau}{12})$, obtained by the discretization over τ. The use of unit symmetric window $w(n)$ of length N is inherently assumed. The procedure for numerical calculation of (9), assuming fixed point t follows:

Step 1: Calculate the set of signals $x_{n_1}(n), x_{p_1}(n)$, $x_{n_2}(n)$ and $x_{p_2}(n)$ by sampling $x(t-\frac{\tau}{6})$, $x^*(t+\frac{\tau}{6})$, $x^*(t-\frac{\tau}{12})$ i $x(t+\frac{\tau}{12})$ over τ, for fixed instant t. Calculate discrete Fourier transforms: $S_{n_1}(t,k)=DFT\left[x_{n_1}(n)\right]$, $S_{p_1}(t,k)=DFT\left[x_{p_1}(n)\right]$, $S_{n_2}(t,k)=DFT\left[x_{n_2}(n)\right]$ and $S_{p_2}(t,k)=DFT\left[x_{p_2}(n)\right]$, for $-N/2\le k\le N/2-1$.

Step 2: Calculate $W_x(t,k)=DFT\left[x_{n_2}(n)x_{p_2}(n)\right]$ as convolution of the form

$$W_x(t,k)=\sum\nolimits_p S_{p_2}(t,p)S_{n_2}(t,k-p), \tag{21}$$

We assume that $S_{n_1}(t,k)$ and $S_{p_1}(t,k)$ are localized in discrete frequency domain, i.e. that the component centered at frequency k_0 is spread over region $[k_0-L,\ k_0+L]$. Under the assumption of symmetric window, this component is localized in the same region for both considered terms. This means that for each k in (21), region $[k-L,\ k+L]$ is considered. The limits for p in (21) are obtained eliminating k_0 from the system of inequalities $k_0-L\le p\le k_0+L$ and $k_0-L\le k-p\le k_0+L$:

$$k/2-L\le p\le k/2+L. \tag{22}$$

Component being centered at k_0 in $S_{n_1}(t,k)$ and $S_{p_1}(t,k)$ is centered at $2k_0$ in resulting $W_x(t,k)$, spreading over region $[2k_0-2L,\ 2k_0+2L]$. The number of frequency points in $W_x(t,k)$ is $2N-1$.

Step 3: Following the previous analysis calculate:

$$R_{x^4}(t,k)=\sum\nolimits_p W_x(t,p)W_x(t,k-p), \tag{23}$$
$$R_{x^8}(t,k)=\sum\nolimits_p R_{x^4}(t,p)R_{x^4}(t,k-p), \tag{24}$$
$$R_{x^{16}}(t,k)=\sum\nolimits_p R_{x^8}(t,p)R_{x^8}(t,k-p). \tag{25}$$

According to the analysis in Step 2, the limits for p in (23) are: $k/2-2L\le p\le k/2+2L$. The signal component in term $R_{x^4}(t,k)$ corresponding to the component at k_0 in $S_{n_1}(t,k)$ and $S_{p_1}(t,k)$, is spread over region $[4k_0-4L,\ 4k_0+4L]$. Convolution $R_{x^4}(t,k)$ is consisted of $4N-3$ frequency samples. Similarly, this component is spread over region $[8k_0-8L,\ 8k_0+8L]$ in term $R_{x^8}(t,k)$, whereas the limits for p in the calculation of (24) are given with $k/2-4L\le p\le k/2+4L$. The resulting convolution $R_{x^8}(t,k)$ is consisted of $8N-7$ samples. For convolution $R_{x^{16}}(t,k)=DFT\left[x^8_{n_2}(n)x^8_{p_2}(n)\right]$ the limits for p read $k/2-8L\le p\le k/2+8L$. The analyzed component is placed in interval $[16k_0-16L,\ 16k_0+16L]$, whereas the number of frequency samples is $16N-14$.

Step 4: Let us calculate the next convolution

$$R_{x^2}(t,k)=\sum\nolimits_p S_{n_1}(t,p)S_{p_1}(t,k-p). \tag{26}$$

Following the continuous-time analysis, the component appearing in $S_{n_1}(t,k)$ appears in the same region and central frequency in $S_{p_1}(t,k)$. The component located in terms $S_{n_2}(t,k)$, and $S_{p_2}(t,k)$ at frequency k_0 is spread over region $[-2k_0-4L,\ -2k_0+4L]$ in terms $S_{n_2}(t,k)$ and $S_{p_2}(t,k)$. Convolution (26) is calculated with following p limits: $k/2-4L\le p\le k/2+4L$. The new region of interest is $[-4k_0-8L,\ -4k_0+8L]$, and the resulting number of points is $2N-1$.

Step 5: The resulting TFR is finally obtained as

$$PD(t,k)=\sum\nolimits_p R_{x^2}(t,p)R_{x^{16}}(t,k-p). \tag{27}$$

It is important to note that the terms order in convolution is crucial as obtained regions for $R_{x^2}(t,p)$ and $R_{x^{16}}(t,k-p)$ differ. Following the results presented in previous steps, the resulting component is spread over $[16k_0-16L,\ 16k_0+16L]$, whereas p is calculated within limits obtained eliminating the unknown k_0 from inequalities $-4k_0-8L\le p\le -4k_0+8L$ and $16k_0-16L\le k-p\le 16k_0+16L$:

$$-k/3-\lceil 16L/3\rceil\le p\le -k/3+\lceil 16L/3\rceil\,, \tag{28}$$

where $\lceil\cdot\rceil$ denotes the rounding to the nearest greater integer. Previous algorithm is presented assuming that samples $x_{n_1}(n), x_{p_1}(n)$, $x_{n_2}(n)$ i $x_{p_2}(n)$ are obtained by discretization over τ. The calculation of discrete representation $PD(n,k)$ based on $x(n)$ assumes a discretization of $x(t)$ over t following the sampling theorem. Samples not appearing on the discrete axis n, that correspond to continuous-time signals $x(t-\frac{\tau}{6})$, $x^*(t+\frac{\tau}{6})$, $x^*(t-\frac{\tau}{12})$ and $x(t+\frac{\tau}{12})$, are obtained by interpolation based on zero-padding in the frequency domain [1].

IV. Numerical results

Example 1: An FM signal being defined as $x(t)=\exp\left(j(20\sin(6\pi t)+j9cos(8\pi t)+j5cos(10\pi t)\right)$ is considered, for $-1\,s\le t\le 1s$ and sampled with period $\Delta t=0.002$. Four TFRs are calculated: $STFT(t,\Omega)$, Pseudo-$WD(t,\Omega)$, S-method $SM(t,\Omega)$ with $Ld=10$ and $PD(t,\Omega)$ with $L=3$. All transforms are calculated using a Hanning window of length $N=256$ (0.512s). The results shown in Fig. 1 (a)-(d) respectively confirm that $PD(t,\Omega)$ calculated according to the proposed realization preserves

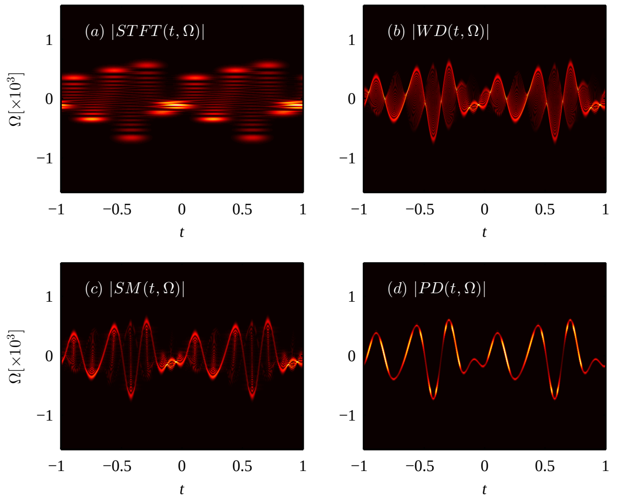

Fig. 1. Comparison of $PD(t, \Omega)$ with various TF representations in the case of a fast varying FM signal.

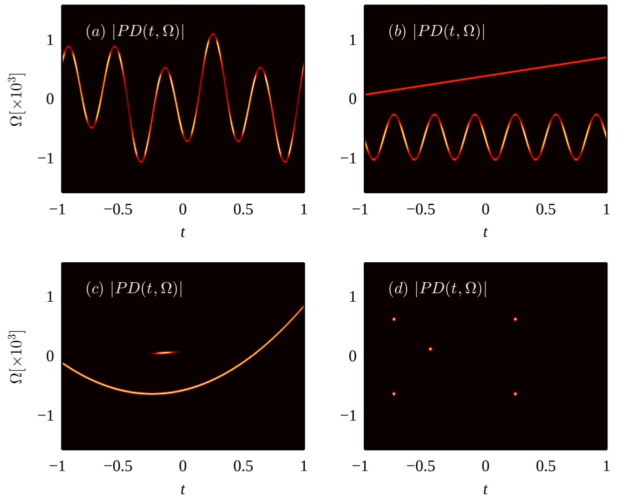

Fig. 2. Representation $PD(t, \Omega)$ calculated for a mono-component (a) and various multi-component signals (b)-(d).

high concentration, while significantly reducing the inner interferences, when compared with other TFRs.

Example 2: Let us observe signals defined for $-1\,s \leq t \leq 1s$, sampled with period $\Delta t = 0.002$. Hanning window of width 0.512s ($N = 256$ samples) is assumed in $PD(t, \Omega)$ calculations. Following signals are considered: (a) mono-component FM signal of the form $x_1(t) = \exp\left(j50\cos(5\pi t) + j50\sin(2\pi t)\right)$; (b) two-component FM signal consisted of one sinusoidally modulated and one LFM component, i.e. $x_2(t) = \exp\left(-j(20\sin(6\pi t) - 200\pi t)\right) + \exp\left(j50\pi t^2 + j100\pi t\right)$; (c) two-component signal consisted of third order polynomial phase signal (PPS) and LFM signal with Gaussian amplitude: $x_3(t) = \exp\left(j100\pi t^3 - j200\pi t\right) + \exp\left(-10(t-0.1)^2\right) \times \exp\left(j20\pi(t+0.4)^2\right)$; and (d) five-component signal consisted of stationary signals having Gaussian amplitudes, defined as follows: $x_4(t) = \sum_{i=1}^{5} \exp\left(-500(t+a_i)^2\right) \exp\left(jb_i\pi(t+c_i)\right)$, with $a_i = [0.2, -0.5, 0.5, 0.5, -0.5]$, $b_i = [40, 200, 200, -200, 200]$ and $c_i = [0.2, -0.5, 0.5, 0.5, 0.5]$, for $i = 1, ..., 5$.

Calculated representations $PD(n, k)$ are shown in Fig. 2 (a)-(d), where $L = 3$ is used. One can observe that a high concentration is obtained, and that inner interferences and cross-terms are significantly reduced. The cross-term reduction and a comparison of the proposed realization method with $PD(n, k)$ calculated by definition is shown in Fig. 3 for the case of a two-component signal.

V. Conclusion

The concept of windowed frequency convolutions, being originally proposed in the S-method theoretical framework, is applied in the realization of a higher order TFR, aiming cross-terms reduction in multi-component signals. The proposed realization method preserves high concentration of the considered representation. Presented results imply that the S-method based implementation approach is general and applicable in higher-order TF analysis. Noise influence on the considered TFR is the part of our further research.

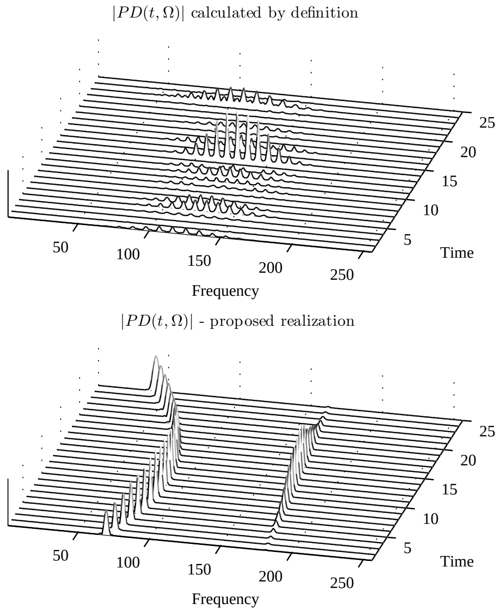

Fig. 3. Cross-terms reduction using the proposed implementation of $PD(t, \Omega)$ in two-component signal (second subplot). Strong cross-terms completely mask the auto-terms in $PD(t, \Omega)$ calculated by definition (first subplot).

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