Overlapping Blocks in Reconstruction of Sparse Images

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***Abstract*—Images are commonly analysed by the discrete cosine transform (DCT) on a number of blocks of smaller size. The blocks are then combined back to the original size image. Since the DCT of blocks have a few nonzero coefficients, the images can be considered as sparse in this transformation domain. The theory of compressive sensing states that some corrupted pixels within blocks can be reconstructed by minimising the blocks sparsity in the DCT domain. Block edges can affect the quality of the reconstruction. In some blocks, a few pixels from an object which mostly belongs to the neighbouring blocks may appear at the edges. Compressive sensing reconstruction algorithm can recognise these pixels as disturbance and perform their false reconstruction in order to minimise the sparsity of the considered block. To overcome this problem, a method with overlapping blocks is proposed. Images are analysed with partially overlapping blocks and then reconstructed using their non-overlapped parts. We have demonstrated the improvements of overlapping blocks on images corrupted with combined noise. A comparison between the reconstructions with non-overlapping and overlapping blocks is presented using the structural similarity index.**

***Keywords*—compressive sensing, image reconstruction, overlapping blocks, gradient algorithm, noisy image**

I. Introduction

An image is said to be sparse if it consists of only few nonzero coefficients in a transformation domain. A sparse image can be reconstructed with a reduced set of pixels. The processing and reconstruction of such images are examined within the theory of compressive sensing (CS) [1]–[9]. The theory of CS is widely used in various applications in the area of digital signal processing, since many real signals are sparse in a certain transformation domain. Numerous reconstruction algorithms for different kinds of signals have been developed within this field. They can be divided in several groups. The algorithm considered in this paper is from the group of algorithms based on the minimisation of the sparsity measure by using the gradient of the L_1-norm [10]. In this algorithm, the image is reconstructed in the spatial (pixels) domain. The corrupted pixels are detected, declared as unavailable (missing) and considered as the minimisation variables. This property makes the algorithm suitable for denoising of corrupted pixels in a noisy environment.

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Common images have a small number of nonzero coefficients in the two-dimensional discrete cosine transform (2D-DCT) space. A reduced set of pixels can be used to reconstruct sparse images. Different reasons can cause that only a reduced set of pixels is available. One reason can be in heavy pixels corruption. Corrupted pixels may be declared as unavailable. Then the image is reconstructed using the CS methods. The impulsive noise is example of such a disturbance. It can appear due to analog to digital conversion errors, communication errors, dead pixels in image acquisition equipment, etc. Here we will consider a form of impulsive noise known as the salt and pepper noise with an addition of noise whose values are within the range of the original image pixels.

The paper is organised as follows. In Section II, theoretical background of sparse signal processing is presented. In Section III, the reconstruction algorithm is introduced. Section IV explains the idea of adding the overlapping block step. In Section V, the results and comparison using the structural similarity index is shown. In Section VI, conclusions are presented.

II. Theoretical Background

Let us consider a grayscale image $I(m,n)$ of size $N \times N$. Based on the JPEG standard, we will split the image into blocks of size $B \times B$. We will assume that a image block starting at pixel (m_0, n_0) is defined as

$$x(m,n) = I(m_0+m, n_0+n), \quad m,n = 0,1,\dots,B-1. \quad (1)$$

Its 2D-DCT representation is denoted by $X(k,l)$. The vector notations of the image blocks and their 2D-DCT are

$$\mathbf{x} = \mathbf{\Psi}\mathbf{X} \text{ and } \mathbf{X} = \mathbf{\Phi}\mathbf{x} \quad (2)$$

where $\mathbf{\Psi}$ and $\mathbf{\Phi}$ are the transform and inverse transform matrices with rearranged elements of the 2D-DCT. Vectors $\mathbf{x}$ and $\mathbf{X}$ are obtained by stacking columns of the corresponding block pixels and 2D DCT transform.

The sparsity of one image block is $K < (B \times B)$. From the compressive sensing theory we know that, if an image is of a sparsity K, it can be reconstructed from less than $B \times B$ pixels/measurements. The measurements are denoted by $\mathbf{y}$

$$\mathbf{y} = [x(m_1,n_1), x(m_2,n_2), ..., x(m_M,n_M)] \quad (3)$$

where M is the number of pixels/measurements used in a block. The relation between the values is $K < M < B \times B$. Our goal, in the sparse signal processing sense, is to reconstruct an image using the available pixels/measurements by minimising the sparsity, i.e.

$$\min \|\mathbf{X}\|_1 \qquad \text{subject to} \quad \mathbf{y} = \mathbf{A}\mathbf{X} \tag{4}$$

where $\mathbf{A}$ is an $M \times N$ measurement matrix obtained from matrix $\mathbf{\Psi}$ by selecting rows that correspond to the available pixels. The measurements are the available (uncorrupted) pixels in the image

$$y(i) = x(m_i, n_i). \tag{5}$$

Positions of the available pixels/measurements are $(m_i, n_i) \in \mathbb{M} = \{(m_1, n_1), (m_2, n_2), ..., (m_M, n_M)\}$. If we have an 8-bit $B \times B$ image block, corrupted with salt and pepper noise, the block can be then written as

$$x_a^{(0)}(m,n) = \begin{cases} x(m,n), & \text{for } (m,n) \in \mathbb{M} \\ 0 \text{ (or } 255), & \text{elsewhere} \end{cases} \tag{6}$$

where 0 and 255 are salt and pepper noise. If an uniform noise is used then the values are between 0 and 255, In the next section we will present an algorithm used for the recovery of the noisy pixels.

III. Reconstruction Algorithm

The algorithm was introduced in [10], [11]. It is based on the minimisation of the gradient of corrupted pixels. Consider a corrupted image block as presented in equation (6). In the initial stage, we add an arbitrary value $\pm\Delta$ to the corrupted pixels

$$\begin{aligned} x_a^+(m,n) &= x^{(p)}(m,n) + \Delta\delta(m - m_i, n - n_i) \\ x_a^-(m,n) &= x^{(p)}(m,n) - \Delta\delta(m - m_i, n - n_i) \end{aligned} \tag{7}$$

where p is the iteration index. In the initial stage $p = 0$.

The arbitrary value Δ is usually the maximal uncorrupted pixel value, i.e. $\Delta = \max_{m,n}(\mathbf{y})$. For the corrupted pixels at positions $(m_i, n_i) \notin \mathbb{M}$ the gradient value is estimated as

$$g(m_i, n_i) = \frac{1}{2\Delta}\left(\left\|\mathbf{X}_a^+\right\|_1 - \left\|\mathbf{X}_a^-\right\|_1 \right) \tag{8}$$

where $\mathbf{X}_a^\pm$ are the 2D-DCT domain values of the signals in (7). Note that the gradient value for the uncorrupted pixels will be zero. Based on the gradient value, the corrupted pixel $x(m_i, n_i)$ is updated. Each corrupted pixel value is changed in the direction opposite of the gradient for a step μ

$$x_a^{(p)}(m_i, n_i) = x_a^{(p-1)}(m_i, n_i) - \mu g(m_i, n_i). \tag{9}$$

Because of the shape of the gradient and norm-one sparsity measure, when the values are close to the true signal values, they will oscillate around the solution. The oscillations are proportional to the step size. When the oscillation is detected, the step sizes Δ and μ are reduced. These new parameters continue approach to the true signal values until a new precision is reached. The procedure is repeated until the desired reconstruction accuracy is achieved. One of the stopping criterion for the algorithm can be if the change in two successive iteration is smaller than some desired accuracy.

Note that only corrupted pixels (which are changed during reconstruction steps) contribute to this change. This is the basic reconstruction algorithm when the positions of the corrupted pixels are known (if, for example, the pixels are distinguishably corrupted).

If other noise types are used, then an additional step is proposed in [12], [13]. Since the positions of the corrupted pixels are unknown, we repeat Eq. (7) and (8) for all pixels (corrupted and uncorrupted). Each time we take the pixel with the largest gradient, reconstruct it and eliminate it from the array of possible values. This will be repeated until the error of two successful iterations is below an acceptable level.

IV. Overlapping blocks

Let us consider an image of size $N \times N$ and that we split the image in number of blocks of size $B \times B$. Each block has M available/uncorrupted pixels. Within blocks we have corrupted pixels with salt and pepper noise and a uniform noise, whose values are similar to the uncorrupted pixels.

Assume that there are a few uncorrupted pixels at the edges which are not of the same or similar value as the other pixels in the block. These pixels mainly belong to the object of a neighbouring block. The compressive sensing theory looks for the sparsest possible solution as the reconstruction result. The sparsest solution of the considered block would be obtained by taking the uncorrupted pixels (which are part of the neighbouring object) as the corrupted ones, since these pixels significantly differ from the majority of the other uncorrupted pixels in that block. In this sense, a method will be falsely reconstruct these pixels as the pixels of similar values to the other pixels within the block.

To overcome this problem, we introduce the overlapping blocks. The idea is to take a bigger block to analyse the objects in surrounding blocks. Then we reconstruct the block and use only a smaller central part of the analysed block in the final image reconstruction. The method of overlapping blocks is suitable as an addition to the algorithms which are based on the detection and reconstruction of the noisy pixels itself, not the transformation domain coefficients.

As an illustration, let say that the block for analysis is of size $B \times B = 32 \times 32$. The size $B_o \times B_o$ will denote the size of the part which will be used in the final reconstruction. Obviously it must hold that $B_o < B$. We assume that we will use the central block of size $B_o \times B_o = 16 \times 16$. Illustration of this kind of blocks is presented in Fig. 1. Bigger blocks represent the blocks for analysis, which are of size $B \times B$, and smaller blocks are the blocks for final reconstruction. They are of size $B_o \times B_o$. Note that for the blocks which are at the edges of the whole image, we use the reconstruction from the edge analysis blocks.

V. Results

In this section we will present reconstruction results using the method presented in the previous sections. Note that the

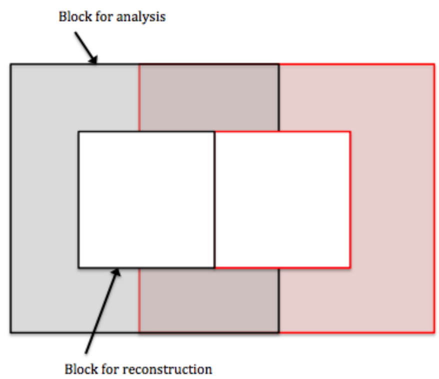

Fig. 1. Illustration of two overlapping blocks. Bigger block is used for the analysis and the smaller one is used for the final reconstruction

idea of overlapping blocks can be considered as an improvement to the reconstruction algorithms for the edges of objects in images. The algorithm was tested using overlapping and non-overlapping blocks. It was tested on a grayscale 512×512 image "Lena" with 50% of salt and pepper noise and 12.5% of random noise. It is considered that 10% of components in each block are nonzero in the 2D-DCT domain. The original image and the noisy image are presented in Fig. 2. The reconstruction without using overlapping blocks is shown in Fig. 3 (top). In the overlapping case the blocks for analysis are of size 32×32 and the part used for the final reconstruction is the central part of size 16×16. The result is shown Fig. 3 (bottom). The images zoomed in to the upper right corner are shown in Fig. 4.

A. Comparison

The comparison between the reconstructions using non-overlapping and overlapping blocks is presented using the localized structural similarity (SSIM) index. The SSIM index is a comparison parameter between two images. It is defined in [14] as

$$\mathrm{SSIM}(\mathbf{x}_o, \mathbf{x}_r) = \frac{(2\mu_{x_o}\mu_{x_r} + c_1)(2\sigma_{x_o x_r} + c_2)}{(\mu_{x_o}^2 + \mu_{x_r}^2 + c_1)(\sigma_{x_o}^2 + \sigma_{x_r}^2 + c_2)} \quad (10)$$

where $\mathbf{x}_o, \mathbf{x}_r$ are the original and the reconstructed image, respectively. The values μ_{x_o}, μ_{x_r} are the mean values of the images, $\sigma_{x_o x_r}$ is the covariance between the two considered images, and $\sigma_{x_o}^2$, $\sigma_{x_r}^2$ are the variances of the two images. The values c_1, c_2 are used as stabilisation variables. If SSIM index is close to 1 the images are similar, if it is close to 0 they are not similar. The SSIM index of the zoomed images from Fig. 4 are shown in Fig. 5.

VI. Conclusions

A method for improving the reconstruction of noisy images using overlapping blocks is proposed. It is an improvement of the methods for reconstruction algorithms which are based on the detection of the corrupted pixels in spatial domain. The reconstruction of the images using non-overlapping and overlapping blocks is shown. The use of overlapping blocks improved results in the denoising of images.

Original image

Noisy image

Fig. 2. Original image (top); Noisy image (bottom) used for the reconstruction

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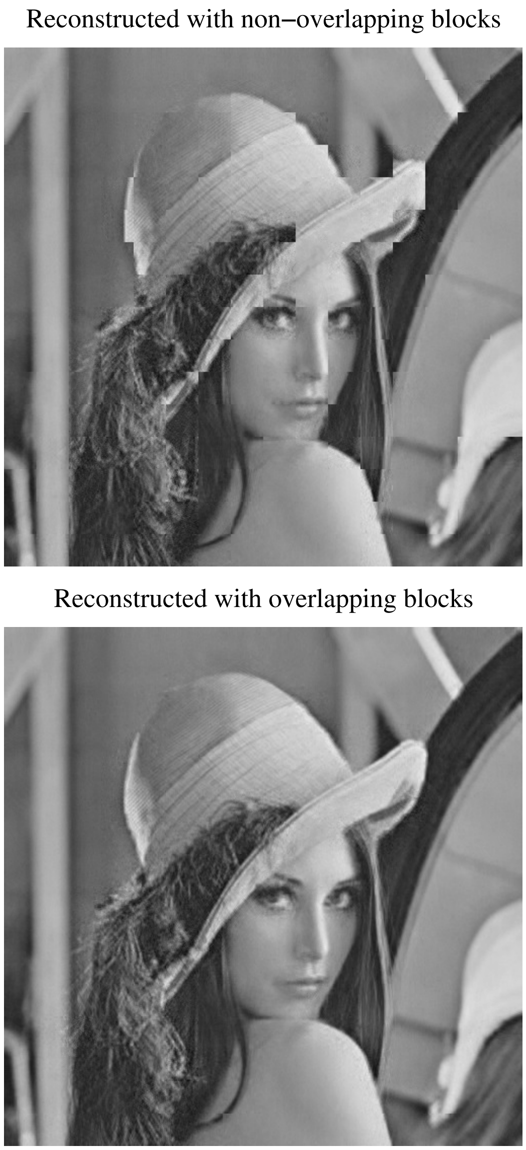

Fig. 3. Reconstructed images: with 32×32 non-overlapping blocks (top); with overlapping blocks (bottom)

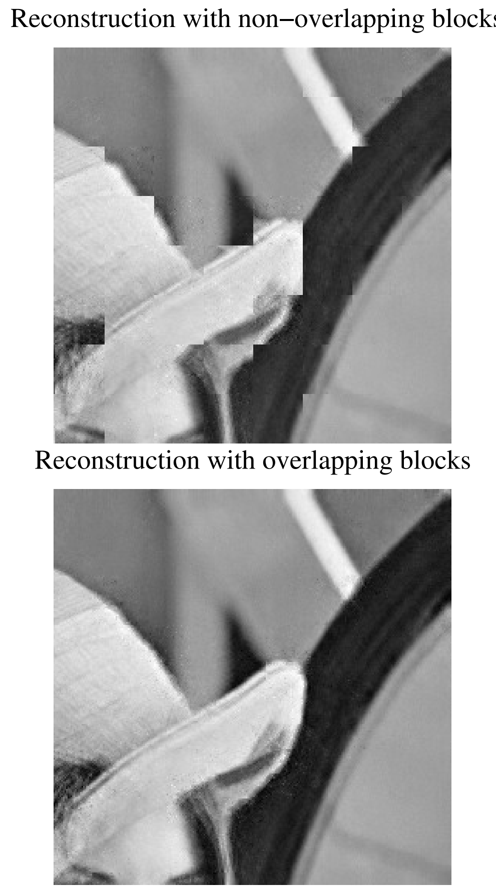

Fig. 4. Zoomed reconstructed images: with 32×32 non-overlapping blocks (top); with overlapping blocks (bottom)

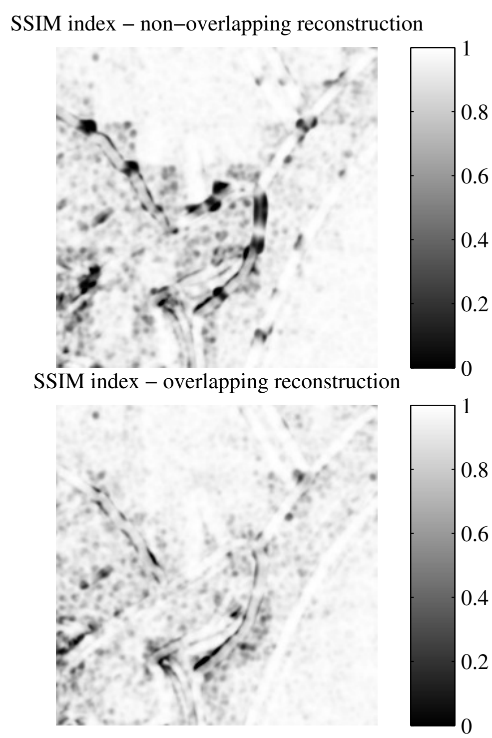

Fig. 5. SSIM index of the zoomed reconstructed images: with 32×32 non-overlapping blocks (top); with overlapping blocks (bottom)

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