

# Time-Frequency Signal Analysis and Processing

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# Time-Frequency Signal Analysis and Processing

## A Comprehensive Reference

Edited by

**Boualem Boashash**



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Academic Press is an imprint of Elsevier



# Preface to the First Edition

Time-Frequency Signal Analysis and Processing (TFSAP) is a collection of theory and algorithms used for analysis and processing of nonstationary signals, as found in a wide range of applications including telecommunications, radar, and biomedical engineering. This book brings together the main knowledge of TFSAP, from theory to applications, in a user-friendly reference suitable for both expert and nonexpert readers.

The **contents** of the book include:

1. a *comprehensive tutorial introduction* to TFSAP, accessible to anyone who has taken a first course in signals and systems;
2. more specialized theory and algorithms, concisely presented by some of the leading authorities on the respective topics; and
3. studies of *key applications*, written by leading researchers, showing how to use TFSAP methods to solve practical problems.

The **motivation** for producing this book was twofold:

- My original and widely used decade-old tutorial on TFSAP [1] needed updating in two respects. First, some of the advances of the last decade are sufficiently fundamental to warrant inclusion in an introductory treatment, while others are sufficiently important to demand coverage in any comprehensive review of TFSAP. Second, new applications have widened the range of disciplines interested in TFSAP, and thus reduced the common background knowledge that may be expected of readers. **Part I** of this book addresses these needs.
- The need for a standard language of discourse became apparent in 1990 while I was editing the 23 contributions to the first comprehensive book in the field [2]. These seminal contributions to TFSAP led to further developments throughout the 1990s, including some significant advances in practical methods suitable for nonstationary signals including instantaneous frequency estimation [3]. These efforts continued apace as this book was being written. Such rapid progress produced a variety of new terminologies and notations that were in need of standardization and inclusion in an updated reference book.

The **organization** of this book uses five parts, each part including several chapters, and each chapter comprising several articles. **Part I** introduces the basic concepts while **Parts II–V** cover more advanced or specialized areas.

**Part I** defines and explains the basic concepts of TFSAP, intuitively derives a variety of well-known time-frequency distributions (TFDs), and then reduces them to a common form. This leads to the general treatment of quadratic TFDs in **Chapter 3**, which should be regarded as the core of the book and as a prerequisite for the later chapters.

**Part II** gives more details on some fundamental topics of TFSAP, such as TFD design and signal analysis in the  $(t, f)$  plane.

**Part III** describes specialized techniques used in implementation, measurement, and enhancement of TFDs.

**Part IV** presents the key statistical techniques for TFSAP of noisy signals, including a full treatment of detection and classification methods.

**Part V** describes a representative selection of TFSAP applications, encompassing telecommunications, radar, sonar, power generation, image quality, automotive applications, machine condition monitoring, and biomedical engineering.

Usability is enhanced by an updated *consolidated bibliography* (alphabetical by author) and a *two-level index* (which also serves as a dictionary of abbreviations).

Under the standard review procedure used for this book, each article had two (usually external) reviewers concentrating on scientific rigor and accuracy, plus two anonymous internal reviewers concentrating on clarity and consistency.

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## ACKNOWLEDGMENTS

**Acknowledgments** are due to a number of people who made possible the completion of this book. Foremost among them are my two sons, who aided me to continue this work during and after my wife's final illness, thus contributing to my sense of balance and purpose during this difficult period. I thank all authors and reviewers, and the organizers of the Special Sessions on TFSAP at ISSPA conferences, for their expertise, timely effort, and professionalism, and for facilitating the exchange of ideas between contributors to this book. I thank my research students and the SPRC staff for valuable assistance. In particular, Gavin Putland assisted with the technical editing of portions of Part I and was responsible for the final mix-down of the authors' L<sup>A</sup>T<sub>E</sub>X™ and PostScript™ files.

**Boualem Boashash, Editor**

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## REFERENCES

- [1] B. Boashash, "Time-Frequency Signal Analysis", in: S. Haykin (Ed.), *Advances in Spectrum Analysis and Array Processing*, vol. 1, Prentice-Hall, Englewood Cliffs, NJ, 1991, pp. 418-517 (Chapter 9).
- [2] B. Boashash (Ed.), "Time-Frequency Signal Analysis: Methods and Applications", Longman-Cheshire/Wiley, Melbourne/New York, 1992.
- [3] B. Boashash, "Estimating and interpreting the instantaneous frequency of a signal – Part 1: Fundamentals; Part 2: Algorithms and applications", *Proc. IEEE*, vol. 80, pp. 519-568, April 1992.

# Preface to the Second Edition

In the decade after the publication of the first edition of this book, advances in the field of time-frequency signal analysis and processing accumulated to an extent that called for an expanded and updated edition. This second edition contains:

1. over 30 new sections by leading experts in their respective fields; covering new time-frequency  $(t, f)$  methods, techniques, algorithms, and applications;
2. new tutorial sections on AM/FM decomposition of signals (Section 1.4), time-scale and wavelet transforms (Section 2.7.6), time-frequency distributions (TFDs) vs. time-scale methods (Section 2.7.7), high-resolution TFDs (Section 3.3.4), positivity of  $(t, f)$  distributions (Section 3.4), Ambiguity function and Radar (Section 3.5); thus making Chapter 3 the most important;
3. a new chapter on “Getting Started” in  $(t, f)$  computation using the original  $(t, f)$  toolbox updated for this second Edition (Chapter 17);
4. a new introductory chapter (numbered  $\mathcal{I}$  to minimize numbering changes), written at a more intuitive level, making the book accessible to a wider readership reflecting the widening use of the  $(t, f)$  approach; and
5. a new chapter on  $(t, f)$  methods in neuroscience. This emerging field of  $(t, f)$  application is the subject of the new Chapter 16, containing six new sections; these concern the assessment of brain EEG and ECG abnormalities (Section 16.1),  $(t, f)$  modeling of EEG signals (Section 16.2), EEG  $(t, f)$  features for classification (Section 16.3), brain networks analysis (Section 16.4), EEG  $(t, f)$  analysis using empirical mode decomposition (Section 16.5), and perspectives for further advances (Section 16.6).

Among the new sections, four relate  $(t, f)$  methods to time-scale methods (Section 4.1), systems (Section 4.9), energy (Section 4.10) and the EMD with the Hilbert spectrum (Section 4.12); four concern directional kernels (Section 5.9), adaptive directional kernels (Section 5.10), the modified S-transform (Section 5.11) and TFDs positivity (Section 5.12). Other new topics include memory-efficient computation of TFDs (Section 6.6), compressive sensing (Section 7.6), signal complexity (Section 7.7), neural networks for  $(t, f)$  localization (Section 7.8), the improvement of  $(t, f)$  representations using  $(t, f)$  post-processing (Section 7.5), introducing multichannel/multisensor  $(t, f)$  processing (Section 8.1), audio source separation (Section 8.6), cyclostationary signals (Section 9.6), extraction of signal components (Section 10.6), polynomial-phase parameter estimation (Section 10.7), IF sequential Bayesian estimation (Section 10.8), subspace noise filtering (Section 11.5), speech enhancement (Section 11.6), generalizing one-dimensional concepts in matched filtering (Section 12.5),  $(t, f)$  feature definition (Section 12.6), modulation parameter estimation (Section 13.5), sparse TFDs in geophysics (Section 14.6), audio signals’  $(t, f)$  characteristics (Section 14.7), using  $(t, f)$  features to improve medical diagnosis (Section 15.5), and structural condition monitoring (Section 15.7).

More information on the scope of new and existing chapter/sections appears in the table of contents and chapter overviews.

The  $(t, f)$  toolbox package (available at the Elsevier website) complements the other chapters of this book. It describes in a simple way the most important algorithms presented in the book and makes them ready for use via an easy practical graphical user interface (GUI). With the aid of Chapter 17, new users can start using the algorithms on simulated and real examples, compare with the results presented in the

book, and insert the algorithms in their own applications. Source code will be made available. Readers can use the package as an introduction to the field (in combination with the tutorial in [Chapter 17](#)), or as a set of MATLAB™-compatible commands that can be accessed from the command line or called from other programs, or as source code that can be adapted for particular applications.

The  $(t,f)$  toolbox TFSAP is a recent upgrade to the package formerly known as TFSA, the first package in its field, which was released publicly in 1987 at the inaugural ISSPA conference in Brisbane, Australia, and frequently upgraded thereafter. It was originally purchased by several hundred universities and research institutions throughout the world, and also made available free of charge as part of evaluation agreements.

In addition to new material, this second edition also updates the text and references of existing sections, ensures more consistent use of common terminology and notations, and enriches the cross-referencing between the various parts of the book. The result is a comprehensive and connected presentation of standard and novel topics in  $(t,f)$  signal analysis and processing. My hope is that it achieves the best possible outcome, within the constraints of a single volume, in equipping modern researchers and engineers with the information and tools necessary to undertake any study, research, or development project in this important scientific field.

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## ACKNOWLEDGMENTS

I thank several colleagues, postdocs, and research assistants who provided much needed help to complete this project. First, Dr Gavin Putland and Ms Ayat Salim assisted with L<sup>A</sup>T<sub>E</sub>X editing of the sections in the book and all related tasks. In addition, several other colleagues contributed to the technical reviews of individual chapters or sections; they include: S. Ouelha, N. A. Khan, E. Sejdic, S. Touati, S. Ali, A. Aïssa-El-Bey, G. Azemi, J. O’Toole, S. Aviyente, B. Jawad, N. Stevenson, H. Hassanpour, A. Saleem, S. Dong, A. Zuri-Sha’ameri, M. A. Awal, A. Ouahabi and M. Alsad. In addition, all individual sections of this book were technically reviewed by at least two experts in the field, as shown in the first page of each section.

Publication of this second edition was facilitated by several grants from Qatar Foundation. In particular, the work in the contributions in [Sections 4.1, 11.6, 13.5, 14.7, and 15.6](#) and the updates to [Sections 7.1, 7.9, 14.4, and 14.5](#), was funded by Qatar National Research Fund, grant number NPRP 6-680-2-282. The work in [Sections 5.11, 6.6, 7.7, 10.6, 11.5, 12.5, 15.5, 16.2, 16.3, 16.4, and 16.6](#), and the updates to [Chapters 1 and 2](#) and [Sections 4.6, 4.12, 5.4, 5.5, 5.7, 5.12, 6.1, 6.5, 7.4, 8.1, 10.3, 10.4, 10.5, 11.2, 11.4, 15.4 and 16.1](#) was funded by Qatar National Research Fund, grant number NPRP 6-885-2-364. The work in [Chapter 17, Sections 5.9, 5.10, 7.5, and 12.6](#), and the updates to [Chapter 3](#), was funded by Qatar National Research Fund, grant number NPRP 4-1303-2-517. The work reported in [Section 16.6](#) was funded by the Australian Research Council and the National Health & Medical Research Council, Australia.

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Academic Press is an imprint of Elsevier  
125 London Wall, London, EC2Y 5AS, UK  
525 B Street, Suite 1800, San Diego, CA 92101-4495, USA  
225 Wyman Street, Waltham, MA 02451, USA  
The Boulevard, Langford Lane, Kidlington, Oxford OX5 1GB, UK

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#### Library of Congress Cataloging-in-Publication Data

A catalog record for this book is available from the Library of Congress

#### British Library Cataloguing in Publication Data

A catalogue record for this book is available from the British Library

ISBN:978-0-12-398499-9

For information on all Academic Press publications  
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Printed and bound in the United Kingdom



*Publisher:* Todd Green  
*Acquisitions Editor:* Tim Pitts  
*Editorial Project Manager:* Charlie Kent  
*Production Project Manager:* Lisa Jones  
*Designer:* Greg Harris

# Book Standard Notations

Symbols frequently used in this book are listed below in alphabetical order, with Latin (English) letters before Greek letters. The meaning in the list below should be assumed unless the symbol is otherwise defined in context.

## Latin

$A(f)$	amplitude spectrum
$a(t)$	instantaneous amplitude (real envelope), e.g., of $s(t) = a(t) \cos \phi(t)$ or $z(t) = a(t)e^{j\phi(t)}$
$A_z(\nu, \tau)$	symmetrical ambiguity function (SAF) of $z(t)$ $= \int_{-\infty}^{\infty} z(t + \frac{\tau}{2})z^*(t - \frac{\tau}{2})e^{-j2\pi\nu t} dt$
$\mathcal{A}_z(\nu, \tau)$	filtered ambiguity function of $z(t)$ $= g(\nu, \tau)A_z(\nu, \tau)$
$B$	bandwidth of signal
$\text{DFT}_{n \rightarrow k}\{x[n]\}$	discrete Fourier transform of $x[n]$ , (from $n$ to $k$ domain)
$\text{DTFT}_{n \rightarrow f}\{x[n]\}$	discrete-time Fourier transform of $x[n]$ , to $f$ domain
$f$	frequency
$f_i(t)$	instantaneous frequency
$\mathcal{F}_{t \rightarrow f}\{x(t)\}$	Fourier transform (FT) of $x(t)$ , to $f$ domain
$\mathcal{F}_{f \rightarrow t}^{-1}\{X(f)\}$	inverse Fourier transform (IFT) of $X(f)$ , back to $t$ domain
$F_z^w(t, f)$	short-time Fourier transform (STFT) of $z(t)$ with window $w(t)$
$\mathbb{F}^\alpha$	fractional FT with parameter $\alpha$
$F_\nu$	frequency-shift operator: $(F_\nu s)(t) = e^{j2\pi\nu t}s(t)$
$\mathcal{G}[l, k]$	discrete Doppler-frequency kernel
$g[l, m]$	discrete Doppler-lag kernel
$G[n, m]$	discrete time-lag kernel
$G(t, \tau)$	time-lag kernel $= \int_{-\infty}^{\infty} g(\nu, \tau)e^{j2\pi\nu t} d\nu$
$\mathcal{G}(\nu, f)$	Doppler-frequency kernel $= \int_{-\infty}^{\infty} g(\nu, \tau)e^{-j2\pi f \tau} d\tau$
$g(\nu, \tau)$	Doppler-lag kernel
$g_\theta(\nu, \tau)$	multi-directional Doppler-lag kernel with set of signal direction angles $\theta$
$H$	operator of system or channel
$\mathcal{H}\{s(t)\}$	Hilbert transform of real signal $s(t)$
$\text{Im}\{.\}$ or $\Im\{.\}$	imaginary part of $\{.\}$

$k$	discrete frequency (in bins)
$K_z(t, \tau)$	instantaneous auto-correlation function (IAF) of $z(t)$ $= z(t + \frac{\tau}{2})z^*(t - \frac{\tau}{2})$
$k_z(\nu, f)$	spectral correlation function (SCF) of $z(t)$ $= Z(f + \frac{\nu}{2})Z^*(f - \frac{\nu}{2})$ , where $Z(f) = \mathcal{F}_{t \rightarrow f}\{z(t)\}$
$\mathbf{K}_{zz}(t, f)$	spatial instantaneous autocorrelation function (SIAF)
$l$	discrete Doppler (frequency shift)
$m$	discrete lag (delay, time shift)
$n$	discrete time (in samples)
$\text{Re}\{.\}$ or $\Re\{.\}$	real part of $\{.\}$
$\text{rect}(x)$	function equal to 1 for $-\frac{1}{2} \leq x \leq \frac{1}{2}$ , and 0 elsewhere
$R_z(t, \tau)$	smoothed instantaneous autocorrelation function (IAF) of $z(t)$ $= G(t, \tau) * K_z(t, \tau)$
$\mathcal{R}_z(t, \tau)$	time-varying autocorrelation of random process $z(t)$
$\mathcal{R}_z(\tau)$	autocorrelation of wide-sense stationary random process $z(t)$
$\text{sinc } x$	$\frac{1}{\pi x} \sin(\pi x)$
$S_z(t, f)$	evolutionary (Wigner-Ville) spectrum of random process $z(t)$ $= \mathcal{F}_{\tau \rightarrow f}\{\mathcal{R}_z(t, \tau)\}$
$S_z(f)$	power spectral density of wide-sense stationary process $z(t)$ $= \mathcal{F}_{\tau \rightarrow f}\{\mathcal{R}_z(\tau)\}$
$S_z^{(H)}(t, f)$	sonograph (sonogram) of $z(t)$ with filter $H(f)$
$S_z^w(t, f)$	spectrogram (squared STFT) of $z(t)$ with window $w(t)$
$T$	duration of signal
$t$	time
$T_\tau$	time-shift operator: $(T_\tau s)(t) = s(t - \tau)$
$w(t)$	window function, or white noise
$W_z(t, f)$	Wigner-Ville distribution (WVD) of $z(t)$ $= \int_{-\infty}^{\infty} z(t + \frac{\tau}{2})z^*(t - \frac{\tau}{2})e^{-j2\pi f\tau} d\tau$
$\text{WT}_x(t, a)$	Wavelet transform of signal $x(t)$ $= \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(u)h^*\left(\frac{u-t}{a}\right) du$



*Greek*

$\gamma[n, k]$	discrete time-frequency kernel
$\gamma(t, f)$	time-frequency kernel = $\int_{-\infty}^{\infty} G(t, \tau) e^{-j2\pi f\tau} d\tau$
$\epsilon(t)$	additive noise with mean $\mu_\epsilon$ and variance $\sigma_\epsilon^2$
$\theta(f)$	phase spectrum
$\nu$	Doppler (frequency shift)
$\Pi_T(t)$	rectangular or box function, also written as $\text{rect}(t/T)$ defined as:
	$\text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & \text{if }  t  \leq T/2 \\ 0 & \text{if }  t  > T/2 \end{cases}$
$\rho_z(t, f)$	quadratic time-frequency distribution (TFD) of signal $z(t)$ $= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi\nu(t-u)} g(\nu, \tau) z(u + \frac{\tau}{2}) z^*(u - \frac{\tau}{2}) e^{-j2\pi f\tau} d\nu du d\tau$ where $g(\nu, \tau)$ is the Doppler-lag kernel
$\rho_{zz}(t, f)$	spatial time-frequency distributions
$\tau$	lag (delay, time shift)
$\tau_d(f)$	spectral delay (time delay)
$\tau_g(f)$	group delay
$\phi(t)$	instantaneous phase
$\psi_{d,c}(u)$	discrete wavelet
$\nabla$	partial differential vector operator

*Math symbols*

$*$	convolution in time
$\underset{t}{*}$	2D convolution in both time and frequency
$\underset{(t,f)}{*}$	
$\equiv, \triangleq$ or $:=$	equal by definition, or defined as
$\propto$	proportional to
$\int_{\mathbb{R}}$	$\equiv \int_{-\infty}^{\infty}$
$\int_{\mathbb{R}^2}$	$\equiv \int_{-\infty}^{\infty} \int_{-\infty}^{\infty}$
$\exists$	there exists

# Abbreviations

The index of this book also serves as the main dictionary of abbreviations; the index entry for each abbreviation is a cross-reference to the corresponding unabbreviated entry. The most common abbreviations are listed below.

FSK	frequency-shift keying
AOK	adaptive optimal kernel
AUC	area-under-the-curve
AWVD	adaptive Wigner-Ville distribution
B-S	burst-suppression
BD	B-distribution
BJD	Born-Jordan distribution
BSS	blind source separation
<i>BT</i> product	bandwidth-duration product
CDMA	code-division multiple-access
CFD	central finite-difference
CKD	compact-support-Kernel distribution
CRLB	Cramer-Rao lower bound
CSK	compact-support-kernel
CWD	Choi-Williams distribution
DCT	discrete cosine transform
DGF	directional Gaussian filter
DGT	discrete Gabor transform
DI	Doppler-independent
DOA	direction-of-arrival
DWT	discrete wavelet transform
DWVD	discrete WVD
EEG	electroencephalogram
EMBD	extended modified B-distribution
EMD	empirical mode decomposition
FFT	fast Fourier transform
FM	frequency modulation
FrFT	fractional Fourier transform
FT	Fourier transform

GAF	generalized ambiguity function
GLRT	generalized likelihood ratio test
GPS	global positioning system
GWD	generalized Wigner distribution
HAF	higher-order ambiguity function
HHT	Hilbert-Huang transform
HVS	human visual system
IA	instantaneous amplitude
IAF	instantaneous autocorrelation function
IDGT	discrete Gabor transform (inverse)
IF	instantaneous frequency
IFT	inverse Fourier transform
IMF	intrinsic mode function
IP	instantaneous phase
ISAR	inverse synthetic aperture radar
KLT	Karhunen-Loève transform
LD	Levin distribution
LFM	linear FM
LI	lag-independent
LTV	linear time-varying
MBD	modified B-distribution
MDD	Multi-directional distribution
MDK	Multi-directional kernel
MIMO	multiple-input multiple-output
ML	maximum likelihood
MSE	mean squared error
MST	modified S-transform
NMF	nonnegative matrix factorization
PCA	principal component analysis
PD	phase differentiation
PLV	phase locking value
PPS	polynomial-phase signal
PSD	power spectral density
PSK	phase-shift keying

PWVDs	polynomial WVDs
QMF	quadrature matched filter
QTFRs	quadratic time-frequency representations
RD	Rihaczek distribution
RID	reduced-interference distribution
ROC	receiver operating characteristic
RWT	Radon-Wigner transform
SAR	synthetic aperture radar
SDS	spectral delay shift
SIMO	single-input multiple-output
SISO	single-input single-output
SM	S-method
SNR	signal-to-noise ratio
SS	spread-spectrum
s.t.	such that
ST	S-transform
STFD	spatial TFD
STFT	short-time Fourier transform
SVD	singular-value decomposition
TF	time-frequency
TFD	time-frequency distribution
TFEC	time-frequency energy concentration
TFMF	time-frequency matched filter
TFPF	time-frequency peak filtering
TFRE	time-frequency Rényi entropy
TFR	time-frequency representation
TFSP	time-frequency signal processing
TK	Teager-Kaiser
TVIR	time-varying impulse response
TV-HOS	time-varying higher-order spectra
TVTS	time-varying transfer function
WPT	wavelet packet transform
WSS	wide-sense stationary
WT	wavelet transform
XWVD	cross-Wigner-Ville distribution

*To my sons  
Jamil Mehdi and Issam Alam,  
my wife Nadia,  
my late spouse Theresa,  
and my whole family.*